Compensation of friction and backlash effects in an electrical actuator

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Abstract: In this paper, non-linear observers, based on an estimation of the friction force and the disturbed torque transmitted due to the dead zone, are developed for systems presenting mechanical imperfections such as friction and backlash. Then an adaptive controller using these non-linear observers is presented, to compensate for mechanical disturbances on-line. Simulation and experimental results applied on an electrical actuator are given to support the theoretical demonstrations.

Keywords: friction force, non-linear observer, backlash effects, transmitted torque, electrical actuator, dead zone, adaptive control

NOTATION

\begin{itemize}
  \item \(a\) area contact width (m)
  \item \(F\) global friction torque (N m)
  \item \(F'\) friction torque error (N m)
  \item \(f_0\) dead zone magnitude (rad)
  \item \(J_m\) motor inertia (N m\(^2\))
  \item \(J_s\) load inertia (N m\(^2\))
  \item \(K\) elasticity constant (N m/rad)
  \item \(N_0\) reducer constant
  \item \(w\) non-linear transmitted torque (N m)
  \item \(\bar{w}\) non-linear transmitted torque error (N m)
  \item \(\alpha_0\) Coulomb friction torque (N m)
  \item \(\alpha_1\) stiction torque (N m)
  \item \(\alpha_2\) viscous friction coefficient (N m s/rad)
  \item \(\varepsilon_e\) input position error (rad)
  \item \(\dot{\varepsilon}_e\) input velocity error (rad/s)
  \item \(\varepsilon_s\) output position error (rad)
  \item \(\dot{\varepsilon}_s\) output velocity error (rad/s)
  \item \(\theta_e\) input reducer position (rad)
  \item \(\dot{\theta}_e\) input reducer velocity (rad/s)
  \item \(\ddot{\theta}_e\) input reducer acceleration (rad/s\(^2\))
  \item \(\theta_s\) output reducer position (rad)
  \item \(\dot{\theta}_s\) output reducer velocity (rad/s)
  \item \(\ddot{\theta}_s\) output reducer acceleration (rad/s\(^2\))
  \item \(\theta_{St}\) Stibbeck velocity (rad/s)
  \item \(\mu\) friction parameter
\end{itemize}

1 INTRODUCTION

The presence of non-linearities in mechanical systems make them difficult to control with high accuracy. Among these imperfections are friction, which depends on the relative velocity of the motion, and the backlash phenomenon issued from the dead zone between two involved parts.

Among research dealing with friction effects, Friedland [1] has developed an algorithm in order to estimate the Coulomb friction force. This algorithm is a reduced-order observer containing two non-linear functions, where one corresponds to the Jacobean of the other. A good choice of non-linear function in the observer allows an asymptotic stability of the error. Amin et al. [2] have developed two types of observers: the first considers the friction force as a constant and the second is used to estimate the relative velocity of the motion during the contact. Canudas de Witt et al. [3] proposed a model of friction that includes different effects, such as hysteresis behaviour and the stiction effect. They developed an adaptive control in order to estimate and then compensate for the friction effects.

The presence of the dead zone in mechanical systems introduces an hysteresis phenomenon between the input and the output positions. This describes the backlash phenomenon, which causes non-stable behaviour in the controlled system. Backlash is inherent in mechanical systems, especially when starting the motion, but if it increases due to wear it will disturb the performance of the system. In such a case, compensation for these effects is due to mechanical or control methods. For a long time, mechanical solutions existed to eliminate these
disturber effects by changing all imperfect parts on the system. Some control solutions have also been proposed, e.g. by Brandenburg and Schafter [4], who studied the influence and the partial compensation of simultaneously acting backlash and Coulomb friction in a speed and position control of an elastic two-mass system. Recker et al. [5] and Tao and Kokotovic [6] worked on the adaptive control of systems with backlash. Different mathematical models were proposed, such as that of Tao and Kokotovic [6], who modelled an inverse backlash model based on a hysteresis cycle. Cadiou and M’Sirdi [7] have developed a differentiable model based on the dead zone characteristic.

In this paper, two non-linear observers have been developed, to estimate the friction force and the disturber transmitted torque. After that, an adaptive controller is presented, to compensate for the disturbance effects. In most applications, the friction and the backlash non-linearities could not be accurately known, so only an estimation of these effects could be possible. A mathematical model of imperfections is given, representing an inverse sigmoid to represent the disturber torque observer [8] and Tustin’s model for the case of friction [9]. Simulation and experimental results are presented in this paper, which are applicable to a bench test constructed in the authors’ laboratory, and an important number of mechanical imperfections are given.

2 DEFINITIONS AND MODELLING

2.1 Description of the bench test

The experimental bench test of Fig. 1a corresponds to an electrical actuator, divided into two parts. The first part represents the motor part and is driven by a d.c. motor. The second part describes the reducer part which regroups three important mechanical imperfections.

The first imperfection is static and viscous friction, where coefficients can be changed for different applications by using many brake parts made of aluminium, metal, etc. These coefficients could also be identified approximately by using classical identification algorithms [7], e.g. the recursive least mean squares method.

The second imperfection is backlash and is represented by a variable dead zone from 0 to 24°. Finally, the transmitted motion to the output axis is via a string system with changeable stiffness.

On this bench test, the measure of input and output positions is taken by two incremental coders, as shown in Fig. 1a.

2.2 Estimation of the friction force

In order to explain the origin of the friction force observer, two different modes are studied. The first one is the nearly static mode, which corresponds to the low-velocity variation. The second one corresponds to the dynamic mode, where the velocity variation is important.

2.2.1 Nearly static mode

Consider that during mechanical motion between two surfaces in contact, the pressure distribution $P(x)$ [8] is given by Fig. 2b. In this case, the distribution is chosen as a half-ellipse in the plane and is limited by $-\alpha$ and $+\alpha$, where $\alpha$ is a positive value that defines the maximum of the deflection [3]. The maximum pressure contact acts at the centre and decreases progressively with the width contact.

The easiest representation of this last distribution could be formulated as follows:

$$P(x) = P_0 \sqrt{1 - \frac{x^2}{\alpha^2}}$$

(1)

For the nearly static mode, the friction force $F_1$ depends on the normal force $N$, so that

$$F_1 = \mu N$$

(2)

where $\mu$ is the friction parameter at low velocities. The normal force $N$ could be deduced by integration of the assumed pressure distribution $P(x)$. The friction force could then be expressed as

$$F_1 = \int_{-\alpha}^{+\alpha} \mu P(x) \, dx$$

(3)
Replacing equation (1) by equation (3) gives
\[ F_1 = \int_{-a}^{a} \frac{\alpha \mu P_0}{1 - \frac{x^2}{a^2}} \, dx \]  
(4)

Putting \( x = a \sin \alpha \) gives \( dx = a \cos \alpha \, d\alpha \), with \( \alpha \in [-\pi/2, \pi/2] \); therefore
\[ F_1 = Pa\mu \int_{-\pi/2}^{\pi/2} \frac{1}{a^2} \cos^2 \alpha \, d\alpha \]  
(5)
\[ F_1 = Pa\mu \int_{-\pi/2}^{\pi/2} \frac{1 + \cos 2\alpha}{2} \, d\alpha \]  
(6)
\[ F_1 = \frac{\pi Pa^2 \mu}{2} \]  
(7)

where \( P_0 \) represents the maximum pressure over the contact surface and is supposed known in this case.

### 2.2.2 Dynamic mode

When \( P(x = \pm a) = 0 \), evolution of the friction force will depend principally on the relative velocity \( \dot{\theta}_s \) between the surfaces in contact:
\[ F_2 = \alpha_2 \dot{\theta}_s \]  
(8)

where \( \alpha_2 \) is the viscous friction coefficient. The global friction force \( F = F_1 + F_2 \) will therefore be expressed for both modes as follows:
\[ F = \frac{\pi Pa^2 \mu}{2} + \alpha_2 \dot{\theta}_s \]  
(9)

### 2.2.3 Friction observer

Consider a low-pass filter formulated as
\[ \dot{\mu} = \frac{1}{(|\theta_{str}|/\lambda)p + 1} \left( \frac{\pi \alpha \lambda}{2a^2 P_0} - \frac{\theta_{str}}{\lambda} k_0 e_s \right) \]  
(10)

where \( \alpha \) is the Coulomb friction, supposed known, \( p \) is the Laplace constant, \( \theta_{str} \) is the Striebeck velocity given in reference [7], \( |\theta_{str}|/\lambda \) is the cut-off frequency, \( \dot{\mu} \) corresponds to the observer of parameter \( \mu \), \( e_s = \theta_s - \theta_d \), where \( \theta_d \) represents the desired output, and \( k_0, \lambda \) are positive constants. The rule of this filter is to cut the high frequencies of \( e_s \), issued as a result of the presence of mechanical imperfections.

The formulation of the \( \mu \) observer and the friction force \( F_2 \) estimator can now be rewritten from equations (9) and (10), which take the same mathematical formulation as their corresponding model, adding the term \( k_0 e_s \), as shown below:
\[ \dot{\mu} = -\frac{\lambda}{|\theta_{str}|} \dot{\mu} + \frac{\pi \alpha \lambda}{2a^2 P_0 |\theta_{str}|} \text{sign}(\dot{\theta}_s) - k_0 e_s \]
\[ \dot{F} = \frac{\pi Pa^2 \mu}{2} + \alpha_2 \dot{\theta}_s \]  
(11)

From equations (11) and (9), the estimated friction error and the observed parameter error \( \mu \) are given as
\[ \dot{\mu} = -\frac{\lambda}{|\theta_{str}|} \dot{\mu} + \rho e \]
\[ \dot{F} = \frac{\pi Pa^2 \mu}{2} - \dot{\mu} \]  
(12)

Putting
\[ G = \frac{\pi Pa^2}{2} \]  
(13)
thus gives the estimated friction error as
\[ \dot{F} = \dot{F} - G\dot{\theta} \]  
(14)

### 2.3 Description of the transmitted torque

Figure 3 shows an approximation of a transmitted torque via a dead zone of magnitude 2\( \theta_0 \), with flexible links. The difference between the input position \( \theta_i \) and the output position \( \theta_o \) of the reducer part is put as \( \Delta \theta \), with \( N_o \) as the reducer constant. The continuous and derivable function, a sigmoid, is used to describe the transmitted torque characteristic inside and outside the dead zone [8], and is easy to implement in the control scheme.
Then the mathematical formulation of this transmitted torque via a dead zone is defined by
\[
C = K \left( \Delta \theta - 4j_0 \frac{1 - e^{-\gamma \Delta \theta}}{1 + e^{-\gamma \Delta \theta}} \right)
\]  
(15)
where \(C\) is the approximate transmitted torque to the load via a flexible axis and a known dead zone. It may be expressed as
\[
C = K \Delta \theta + w
\]  
(21)
where \(K\) is the stiffness of the flexible parts and \(w\) is the disturber torque, described before. Note that
\[
\Delta h = \theta_e - N_0 \theta_s
\]  
(22)
which represents the difference between the input and the output positions of the reducer part. \(N_0\) is the reducer constant.

System (20) can then be expressed as follows:
\[
\begin{align*}
J_f \ddot{\theta}_e + F &= K \theta_e - KN_0 \theta_s + w \\
J_m \ddot{\theta}_s + f_m \dot{\theta}_s + C &= U - K \theta_s + KN_0 \theta_s - w
\end{align*}
\]  
(23)

\[
\begin{align*}
\varepsilon_s &= \theta_s - \theta_s^d \\
\varepsilon_e &= \theta_e - \theta_e^d \\
z &= K \theta_e - KN_0 \theta_s \\
\varepsilon_z &= z - z_d \\
\bar{w} &= w - \bar{w} \\
\bar{F} &= F - \bar{F}
\end{align*}
\]  
(24)
where \(\theta_s^d\) is the desired output position, \(\theta_e^d\) is the desired input position, \(z\) is the difference between the input and the output positions of the reducer part and \(z_d\) is its desired value, \(\bar{w}\) is the estimated disturber torque and \(\bar{w}\) corresponds to the estimated error of the disturber torque. Now, system (23) may be expressed as
\[
\begin{align*}
J_f \ddot{\theta}_e + F &= z + w \\
J_m \ddot{\theta}_s + f_m \dot{\theta}_s + J_f \ddot{\theta}_e + F &= U
\end{align*}
\]  
(25)

The dynamic model of the bench test given in Fig. 1, including the friction and backlash imperfections, is described by the following equations system:
\[
\begin{align*}
J_f \ddot{\theta}_e + F &= C \\
J_m \ddot{\theta}_s + f_m \dot{\theta}_s + C &= U
\end{align*}
\]  
(20)

where \(J_f, \dot{\theta}_e\) are the inertia of the reducer part supposed known and the output reducer acceleration respectively, \(J_m, \dot{\theta}_s, \ddot{\theta}_s\) are the inertia of the motor part, the viscous input friction, which are supposed known, the input reducer acceleration and velocity respectively, \(U\) is the control torque, \(F\) is the friction force, supposed unknown in the actuator and has to be estimated, and \(\theta_e\) and \(\dot{\theta}_e\) are the input and output measured positions of the reducer part respectively. \(C\) represents the transmitted torque to the load via a flexible axis and a known dead zone. It may be expressed as

**Fig. 3** Approximation of transmitted torques
The control scheme to the bench test is given by Fig. 4. C₁ and C₂ are proportional derivative (PD) controller blocs with transient blocs of H₁ and H₂.

In order to linearize the first equation of system (25), the \( z \) and \( w \) values given in system (24) are replaced by estimated values as follows:

\[
J_0 \dot{\theta}_e + F = \varepsilon + z_d + \dot{w} + \ddot{w}
\]  
(26)

For that, \( z_d \) is chosen as

\[
z_d = J_0 \dot{\theta}_e + \dot{F} - \dot{w}
\]  
(27)

By replacing expression (27) in equation (26), the output system equation is given by

\[
J_0 \dot{\varepsilon} + \ddot{F} = \varepsilon + \ddot{w}
\]  
(28)

which is the equation deduced after linearization of the reducer part model.

Now, to linearize the second equation of system (25), the following affectionates are used:

\[
\begin{align*}
\dot{\theta}_e &= \frac{z}{K} + N_0 \dot{\theta}_s \\
\dot{\varepsilon} &= \frac{z}{K} + N_0 \dot{\theta}_s \\
\ddot{\varepsilon} &= \frac{z}{K} + N_0 \dot{\theta}_s
\end{align*}
\]  
(29)

The second equation of system (25) will become

\[
\frac{J_m}{K} \ddot{z} + \frac{f_m}{K} \dot{z} = (J_m N_0 + J_s) \dot{\theta}_s + f_m N_0 \dot{\theta}_s + F = U
\]  
(30)

Thus, the control law \( U \) for global system (20) is expressed as

\[
U = \frac{J_m}{K} \dot{z} + \left( \frac{f_m}{K} + \frac{K_{D1}}{K} \right) \dot{z} + \frac{K_1}{K} \dot{z} + \frac{K_1}{K} \dot{z} + (J_m N_0 + J_s) \dot{\theta}_s + f_m N_0 \dot{\theta}_s + \dot{F}
\]  
(31)

where \( K_p \) and \( K_{D1} \) are the PD constants of the C₁ controller and \( \dot{F} \) the estimated friction force given in Fig. 4.

According to the control scheme of Fig. 4,

\[
\dot{\theta}_e = K_{D2} \varepsilon + K_{P2} \varepsilon
\]  
(32)

where \( K_p \) and \( K_{D2} \) represent the C₂ controller constants of Fig. 4.

By replacing the expression for \( U \) in equation (30), the following equation is obtained:

\[
\frac{J_m}{K} \ddot{z} + \left( \frac{f_m}{K} + \frac{K_{D1}}{K} \right) \dot{z} + \left( \frac{K_1}{K} \right) \dot{z} + (J_m N_0 + J_s - K_{D1} K_{D2} \varepsilon) \varepsilon + (f_m N_0 - K_{P1} K_{D2} - K_{D1} K_{P2} + K_{D1} N_0) \varepsilon + (K_{P1} N_0 - K_{P1} K_{P2}) \varepsilon + \dot{F} = 0
\]  
(33)

This last equation describes the linearization of the motor part of our bench test.

From equations (11) and (9), the estimation errors of the friction parameter \( \mu \) and the friction force are given by the following equations system:

\[
\dot{\mu} = -\lambda \mu + k_0 \varepsilon
\]
\[
\dot{F} = G \mu
\]  
(34)

where

\[
G = \frac{\pi P_o a^2}{2} \quad \text{and} \quad \lambda \frac{\mu}{\lambda \mu}
\]

The torque estimator has the same formulation as its model (18) and is given by the following expression:

\[
\dot{w} = -4K_{J0} \left[ 1 - e^{-\gamma \Delta \theta} \right] \frac{\mu}{\lambda \mu}
\]  
(35)

with \( \gamma \) supposed known. In the case of \( \Delta \theta > 0 \), \( \dot{w} \) will take the following expression:

\[
\dot{w} = -4K_{J0}
\]  
(36)

---

**Fig. 4** Control scheme of the bench test including friction and backlash
after choosing a backlash magnitude model given by
\[
\frac{d\tilde{f}_0}{dt} = 0
\]  
(37)
with \(f_0\) the magnitude constant. Thus, the observer magnitude can be expressed by the output position errors and output velocity errors, as follows:
\[
\frac{d\tilde{f}_0}{dt} = -k_2\dot{e}_s - k_3\dot{e}_s
\]  
(38)
where \(k_2\) and \(k_3\) are positive constants.

The estimation error of the backlash magnitude is given as
\[
\frac{d\tilde{f}_0}{dt} = \frac{d\tilde{f}_0}{dt} - \frac{d\tilde{f}_t}{dt} = k_2\dot{e}_s + k_3\dot{e}_s
\]  
(39)
Thus, the global system will be a combination of all the following equations:
\[
J_m\ddot{e}_s + \frac{f_m}{K} + \frac{K_{D1}}{K}\dot{e}_s + \left(k_1 + \frac{K_{P1}}{K}\right)\dot{e}_s \\
+ (J_mN_0 + J_s - K_{D1}K_{D2})\dot{e}_s \\
+ (f_mN_0 - K_{F1}K_{D2} - K_{D1}K_{D2} + K_{D1}N_0)\dot{e}_s \\
+ (K_{P1}N_0 - K_{P1}K_{P2})\dot{e}_s + \tilde{F} = 0
\]
\[
\frac{d\tilde{f}_0}{dt} = k_2\dot{e}_s + k_3\dot{e}_s
\]
\[
\dot{\tilde{w}} = -4K\tilde{f}_0
\]
\[
\dot{\tilde{u}} = -\dot{\tilde{w}}/K + K_0\dot{e}_s
\]
\[
\tilde{F} = G\tilde{u}
\]  
(40)
which is represented in Fig. 5, with
\[
b_3 = \frac{J_m}{K}
\]
\[
b_2 = \frac{\xi}{K} + \frac{K_{D1}}{K}
\]
\[
b_1 = \frac{\xi}{K} + \frac{K_{P1}}{K}
\]
\[
b_0 = \frac{\xi}{K} + \frac{K_{P1}}{K}
\]
\[
a_3 = K_{D1}K_{D2} - J_mN_0 - J_s
\]
\[
a_2 = K_{P1}K_{D2} + K_{D1}K_{P2} - f_mN_0 - K_{D1}N_0
\]
\[
- \xi(J_mN_0 + J_s - K_{D1}K_{D2})
\]
\[
a_1 = K_{P1}K_{P2} - K_{P1}N_0
\]
\[
- \xi(f_mN_0 - K_{P1}K_{D2} - K_{D1}K_{P2} + K_{D1}N_0)
\]
\[
a_0 = Gk_0 - \xi(K_{P1}N_0 - K_{P1}K_{P2})
\]

Fig. 5 Equivalent scheme of the global system in a closed loop

The characteristic equation of the global system (40) is defined by the following equation:
\[
C_6\tilde{p}^8 + C_7\tilde{p}^7 + C_8\tilde{p}^6 + C_9\tilde{p}^5 + C_{10}\tilde{p}^4 + C_{11}\tilde{p}^3 + C_{12}\tilde{p}^2 + C_1\tilde{p} = 0
\]  
(41)
with
\[
C_8 = 4KJ_1a_3
\]
\[
C_7 = 4KJ_1(\xi + 1)a_2 + 4Kb_3 + k_3a_3
\]
\[
C_6 = 4KJ_1a_0 + a_3(4K\tilde{g}_0 + k_3) + 4K\xi a_2 + a_2k_2 + a_2K\xi
\]
\[
C_5 = 4KJ_1a_0 + a_3(4K\tilde{g}_0 + k_3) + 4K\xi a_2 + a_2k_2 + a_2K\xi
\]
\[
C_4 = 4K(\xi + 1)a_0 + 4Kb_3 + k_3a_3 + 4K(\xi + \xi a_2) + a_3k_3\xi
\]
\[
C_3 = 4K(\xi + 1)a_2 + a_2k_2 + a_2(4K\tilde{g}_0 + k_3) + a_2k_3\xi
\]
\[
C_2 = 4K\tilde{g}_0 a_0 + a_0(4K\tilde{g}_0 + k_3) + k_3\tilde{g}_0
\]
\[
C_1 = k_3\tilde{g}_0 a_0
\]

To define the conditions on the controller constants and on constants \(K_0\), \(K_1\), \(K_2\), \(K_4\), use of the Routh criterium determines the stability limits for the global system to converge to the equilibrium state (\(\dot{e}_s \rightarrow 0\), \(\ddot{e}_s \rightarrow 0\), \(\dot{w} \rightarrow 0\), \(\tilde{F} \rightarrow 0\)).

4 SIMULATION RESULTS

The simulation tests are performed on a mechanical model representative of the bench test of Fig. 1b. The parameters of the model for the simulation tests are
\[
K_{P1} = 15, \quad K_{D1} = 0.5, \quad K_{P2} = 15, \quad K_{D2} = 0.3
\]
\[
K = 1 \text{ N m/rad}, \quad J_m = 0.000 972 \text{ N m}^2\text{rad}
\]
\[
f_m = 0.000 43 \text{ N m s/rad}, \quad J_s = 7.5 \text{ N m}^2\text{rad}
\]
\[
\alpha_0 = 8 \text{ N m}, \quad \alpha_1 = 10.5 \text{ N m}, \quad \alpha_2 = 16 \text{ N m s/rad}
\]
\[
j_0 = 0.1 \text{ rad}, \quad k_1 = 1, \quad k_2 = 0.01, \quad k_3 = 1
\]
and
\[
N_0 = 59
\]
Figure 6a describes the tracking of the output position for a desired output signal $\theta_d(t) = 0.5 \sin(0.2\pi t)$ and a PD controller applied on the system of Fig. 1. Figure 6b represents the output position error before the adaptive compensation of the imperfection effects. It can be seen that the real output position signal is less deformed at the peak area due to the presence of the disturbances. This deformation is compensated after introducing the adaptive compensation, as shown in Fig. 7a. Then, the real output position signal approaches the desired one, with a position error described as in Fig. 7b. This error is especially due to the flexibility effects, which are not considered in the adaptive compensation. Figure 8a describes the tracking of the input signal before compensation for the mechanical imperfection effects. The tracking is very good in the two cases but is clearer in the case after compensation (Fig. 9a). The difference between the error input position tracking is shown in Fig. 8b before compensation and in Fig. 9b. After adding observers, the output tracking error is reduced and is uniform for each period. However, a static error is still present, due to the presence of the flexibility effects in the mechanical system. Figure 10 describes the hysteresis cycle between the input and the output reducer positions. After compensating for the dead zone effect, the width of the cycle is reduced. Knowing that the flexibility effect is still present in the modelling, the relation between the input and output positions is not exactly linear after compensation. Figure 11a represents the control signals before and after compensation. The signal before compensation is not as clear as the one after compensation. This is due to the presence of the imperfections, defined by the disturber torque of Fig. 11b. After compensation, a linear representation of the torque is obtained, which is transmitted via a flexible axis, as shown in Fig. 11b.
Fig. 8  (a) Desired and real input signals before compensation; (b) input position error before compensation

Fig. 9  (a) Desired and real input signals after compensation; (b) input position error after compensation

Fig. 10  Hysteresis backlash behaviour before and after compensation
5 EXPERIMENTAL RESULTS

The experimental tests have been applied on the bench test of Fig. 1a, using the following control parameters:

- \( K_{P1} = 1 \), \( K_{D1} = 0.01 \), \( K_{P2} = 10 \), \( K_{D2} = 5 \)
- \( K = 2 \text{ Nm/rad} \), \( j_0 = 0.48 \text{ rad} \), \( k_1 = 1 \)
- \( k_2 = 0.02 \), \( k_3 = 1 \), \( \alpha_0 = 8 \text{ Nm} \)
- \( \alpha_1 = 10.5 \text{ Nm} \), \( \alpha_2 = 16 \text{ Nm/s/rad} \)

In these tests, the motor reducer was required to move from the initial static output position \( \theta_s(0) = \pi/2 \text{ rad} \) and output velocity \( \dot{\theta}_s(0) = 0 \text{ rad/s} \) to the origin \( \theta_s(0) = 0 \text{ rad}, \dot{\theta}_s(0) = 0 \text{ rad/s} \).

Figure 12 represents the tracking output position before (i.e. the regulation is made by only a PD controller) and after applying the adaptive compensation. The static position error is about 0.32 rad and is compensated after adding the estimators of the undesired dead zone torque and friction force. Figure 13 shows the output velocity signals before and after the adaptive compensation. Therefore, before the compensation case, undesired oscillations around \( \dot{\theta}_s(0) = 0 \text{ rad/s} \) are present and represent the non-linearity effects. These imperfections are compensated after applying the estimated disturber torque and friction force. Finally, the control signals before and after compensating the backlash and friction effects are shown in Fig. 14. In the case after compensation, the control signal is clearer than its equivalent before the compensation due to the adaptive compensation of the disturbance effects of friction and backlash.

6 CONCLUSION

The presence of mechanical imperfections such as friction and backlash in controlled systems make them difficult to control with high accuracy. The mechanical
imperfection effects could be reduced by estimating the necessary disturber torque inside the dead zone and the friction force acting during the motion. After adding these observers into the control law, the undesired nonlinearities can be reduced. For the case of friction estimation, a dynamic model describing the friction force variation as a function of the output system velocity is presented. Then, the friction observer corresponds to a filter bloc, where its input is the output position error and its output is the friction parameter. For the case of backlash, a non-linear and derivable mathematical model for the disturber torque is presented, where the dead zone magnitude approaches a constant value. Estimation of the magnitude variation is observed as a function of the output position error and output velocity error. A good choice of control system parameters allows the convergence of the global system to the original state, as shown in the simulation and experimental results.

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