Second Order Sliding Mode Observer for Estimation of Road Profile

A. Rabhi¹, N.K. M’sirdi¹, L. Fridman² and Y. Delanne³

Abstract—This paper deals with an approach to estimate the road profile, by use of second order sliding mode observer. The method is based on a robust observer designed with a nominal dynamic model of vehicle. The estimation accuracy of observer has been validated experimentally using a trailer equipped with position sensors and accelerometers.

Keywords—Road Profile, Vehicle dynamics, Sliding Modes observer, Robust nonlinear observers.

I. INTRODUCTION

Road profile unevenness through road/vehicle dynamic interaction and vehicle vibration affects safety (Tyre contact forces), ride comfort, energy consumption and wear of tire. Thus, an overview of the road profile (over a wide distance) appears to be necessary to qualify the serviceability of a road pavement. The road profile unevenness is consequently a basic information for road maintenance management systems.

For the purpose of road serviceability, surveillance and road maintenance, several profilometers have been developed. For instance, [1] have proposed a method based on direct measurements of the road roughness. However, some drawbacks of this method and some limitations of its capabilities have been pointed out in [2].

The Road and Bridges Central Laboratory in French (LCPC) has developed a Longitudinal Profile Analyser (LPA) [3]. It is equipped with a laser sensor to measure the elevation of road profile. A profilometer is an instrument used to produce series of numbers related in a well-defined way to the true profile [1]. However, this instrument produces biased and corrupted measures. Other geometrical methods using many sensors (distance sensor, accelerometers...) were also developed [4]. However, these methods depend directly on the sensors reliability and cost. It is worthwhile to mention that these methods do not take into consideration the dynamic behaviour of the vehicle.

In a previous work, M’Sirdi and all [5][7][8] have presented an observer to estimate the road profile by means of sliding mode observers designed from a dynamic modelization of the vehicle. But in the previous method the vehicle rolling velocity is constant and steering angle is assumed zero. For estimation of the road profile, slope and inclination are also neglected. The main contribution is here to extend this observer.

This paper is organized as follows: section 2 deals with the vehicle description and modelling. The design of the second order sliding mode observer is presented in section 3. Some results about the states observation and road profile estimation by means of the proposed method are presented in section 4.

Finally, some remarks and perspectives are given and commented in a concluding section.

II. VEHICLE DYNAMIC MODEL

In literature, many studies deal with vehicle modelling [9][10][11]. The objective may be either comfort analysis or design or increase of safety and maniability of the car. The dynamic equations of the motion of the vehicle body are obtained by applying the fundamental principle of mechanics.

![Diagram](image_url)

Fig. 1. vehicle modele

The system under consideration is a vehicle represented as depicted in figure 1.

This vehicle is composed by a car body, four suspensions and four wheels.

\[ z, \theta \text{ and } \phi \text{ represent the displacements of the vehicle body, roll angle, and pitch angle respectively.} \]

\[ z_i = i+4 \text{ is the displacement of the wheel } i. \]

\[ U = [u_1 \ u_2 \ u_3 \ u_4]^T \] is the vector of the unknown inputs which characterizes the road profile and will be estimated. 

¹LSIS, CNRS UMR 6168, Dom. Univ. St Jérôme, Av Escadrille Normandie-Niemen 13397 Marseille France

²UNAM Dept of Control, Division of Electrical Engineering, Faculty of Engineering, Ciudad Universitaria, Universidad Nacional Autonoma de Mexico, 94510, Mexico, D.F., Mexico

³LCPC Nantes: Division ESAR BP 44341 44 Bouguenais Cedex abdelhamid.rabhi@lsis.org
When considering the vertical displacement along the vertical axis \( z \), the dynamic of the system can be written as:

\[
M \ddot{q} + C \dot{q} + Kq = AU
\]

where \((q, \dot{q})\) represent the velocities and accelerations vector respectively.

\( M \in R^{7 \times 7} \) is the inertia matrix, \( C \in R^{7 \times 7} \) is related to the damping effects, \( K \in R^{7 \times 7} \) is the springs stiffness vector (see Figure 1).

The car body is assumed rigid.

The matrix \( M, C, K \) and \( A \) are defined in appendix.

\( q \in R^7 \) is the coordinates vector defined by:

\[
q = [z_1, z_2, z_3, z_4, \theta, \phi]
\]

III. ESTIMATION OF THE ROAD PROFILE

The dynamical model (1) can be written in the state form as follows:

\[
\begin{align*}
\dot{x}_1 &= q \\
\dot{x}_2 &= x_2 \\
\dot{x}_3 &= x_3 = \ddot{q} = M^{-1}(-C x_2 - K x_1 + AU)
\end{align*}
\]

where the state vector \( x = (x_1, x_2)^T = (q, \dot{q})^T \), and \( y = q \) (\( y \in R^7 \)) is the vector of measured outputs of the system.

\[
y = [z_1, z_2, z_3, z_4, \theta, \phi]^T
\]

Thus, we obtain:

\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= f(x_1, x_2) + \xi
\end{align*}
\]

with

\[
f(x_1, x_2) = M^{-1}(-C x_2 - K x_1)
\]

The unknown input component is

\[
\xi = M^{-1}AU
\]

In order to estimate the state vector \( x \) and to deduce the unknown inputs vector \( U \), we propose the following second order sliding mode observer [15]:

\[
\begin{align*}
\dot{\hat{x}}_1 &= \hat{x}_2 + z_1 \\
\dot{\hat{x}}_2 &= f(t, x_1, \hat{x}_2) + z_2
\end{align*}
\]

where \( \hat{x}_1 \) and \( \hat{x}_2 \) are the state estimations, and the correction variables \( z_1 \) and \( z_2 \) are calculated by the super-twisting algorithm

\[
\begin{align*}
z_1 = \lambda |x_1 - \hat{x}_1|^{1/2} \text{sign}(x_1 - \hat{x}_1) \\
z_2 = \alpha \text{sign}(x_1 - \hat{x}_1)
\end{align*}
\]

The initial moment \( \dot{x}_1 = x_1 \) and \( \dot{x}_2 = 0 \), are taken to ensures observer convergence.

We assume \( x_1 \) available for measurement and we propose the following sliding mode observer:

\[
\begin{align*}
\dot{\hat{x}}_1 &= \hat{x}_2 + \lambda \sqrt{|x_1 - \hat{x}_1|} \text{sign}(x_1 - \hat{x}_1) \\
\dot{\hat{x}}_2 &= f(t, x_1, \hat{x}_2) + \alpha \text{sign}(x_1 - \hat{x}_1)
\end{align*}
\]

where \( \hat{x}_1 \) represent the observed state vector and \( \alpha, \beta \) and \( \lambda \) are the observer gains.

It is important to note that in a first step, input effects on the dynamic are rejected by the proposed observer like a perturbation.

Taking \( \dot{x}_1 = x_1 - \hat{x}_1 \) and \( \dot{x}_2 = x_2 - \hat{x}_2 \) we obtain the equations for the estimation error dynamics

\[
\begin{align*}
\dot{\hat{e}}_1 &= \hat{x}_2 - \lambda |\hat{x}_1|^{1/2} \text{sign}(\hat{x}_1) \\
\dot{\hat{e}}_2 &= F(t, x_1, \hat{x}_2) - \alpha \text{sign}(\hat{x}_1)
\end{align*}
\]

Let us recall that

\[
F(t, x_1, \hat{x}_2) = f(t, x_1, \hat{x}_2) - f(t, x_1, \hat{x}_2) + \xi(t, x_1, x_2)
\]

In our case, the system states are bounded, then the existence of a constant bound \( f^+ \) is ensured such that

\[
|F(t, x_1, \hat{x}_2)| < f^+
\]

holds for any possible \( t, x_1, x_2 \) and \( |\dot{x}_2| \leq 2v_{\text{max}} \).

\( v_{\text{max}} \) and \( x_{\text{max}} \) are defined such that \( \forall t \in R^+ \), \( \forall x_1, x_2 \)

\( |x_1| \leq v_{\text{max}} \) and \( x_1 \leq x_{\text{max}} \).

The state boundedness is true, because the mechanical system (5) is BIBS stable, and the control input \( u \) is bounded.

The maximal possible acceleration in the system is a priori known and it coincides with the bound \( f^+ \).

In order to define the bound \( f^+ \) let us consider the system physical properties.

We have:

\[
\begin{align*}
-\frac{m}{2} I &\leq M \leq \frac{m}{2} I \\
-\frac{\ell}{2} I &\leq C \leq \frac{\ell}{2} I \\
-\frac{K}{2} I &\leq K \leq \frac{K}{2} I
\end{align*}
\]

where \( m, \ell \) and \( K \) are the minimal respective eigenvalues and \( \frac{m}{2}, \frac{\ell}{2} \) and \( \frac{K}{2} \) the maximal ones.

Then we obtain \( \max(M^{-1}) = \frac{1}{m} I \) and \( f^+ \) can be written as

\[
f^+ = \frac{1}{m} (v_{\text{max}}^2 + \bar{v}_{\text{max}}^2)
\]

Let \( \alpha \) and \( \lambda \) satisfy the following inequalities, where \( p \) is some chosen constant, \( 0 < p < 1 \)

\[
\lambda > \frac{\alpha}{2} \frac{p}{2} \frac{(fl + p)}{(1-p)}
\]

Theorem 1: The observer (8), (9) for the system (5) ensures the finite time convergence to estimate the system, i.e. \((\hat{x}_1, \hat{x}_2) \rightarrow (x_1, x_2)\).

The proof is given by Davila and Fridman in [16].

The previous observers ensures that in finite time we have \( \hat{x}_2 = 0 \) then

\[
\dot{\hat{x}}_2 = f(x_1, x_2) - f(x_1, \hat{x}_2) + \xi - z_2 = 0
\]
Let us take a low pass filtering of \( z_2 \) which is defined in equation 8 and 9, then we obtain in the mean average:

\[
\xi = \tau_2
\]  

(17)

Note that \( \tau_2 \) is the filtered version of \( z_2 \).

In order to estimate the elements \( u_i, i = 1...4 \) of the unknown input vector \( U \) and according 7 we can write

\[
\zeta_1 = A_{11}U + B_{11}\dot{U}
\]

with \( \zeta = [\zeta_1 \ 0 \ 0]^T \), and the matrices \( A_{11} \) and \( B_{11} \) given by:

\[
A_{11} = \begin{bmatrix}
\frac{k_{11}}{m_1} & 0 & 0 & 0 \\
0 & \frac{k_{22}}{m_2} & 0 & 0 \\
0 & 0 & \frac{k_{f1}}{m_3} & 0 \\
0 & 0 & 0 & \frac{k_{f2}}{m_4}
\end{bmatrix}, \quad B_{11} = \begin{bmatrix}
\frac{B_{11}}{m_1} & 0 & 0 & 0 \\
0 & \frac{B_{22}}{m_2} & 0 & 0 \\
0 & 0 & \frac{B_{f1}}{m_3} & 0 \\
0 & 0 & 0 & \frac{B_{f2}}{m_4}
\end{bmatrix}
\]

for \( i = 1...4 \) we have

\[
\zeta_{1i} = a_{ii}u_i + b_{ii}\dot{u}_i
\]  

(18)

where \( a_{ii} \) and \( b_{ii} \) are respectively the elements of \( A_{11} \) and \( B_{11} \).

To solve this system we can take an approach simpler that the one in [7] which uses a standard observer.

We can write:

\[
\begin{cases}
\dot{u}_i = g(u_i, \zeta_{1i}) \\
\zeta_{1i} = h(u_i)
\end{cases}
\]  

(19)

with:

\[
g(u_i, \zeta_{1i}) = \frac{1}{b_{ii}}(-a_{ii}u_i + \zeta_{1i})
\]  

(20)

The observer proposed here is the:

\[
\dot{\hat{x}}_i = f(\hat{x}_i, \hat{y}_i) + \lambda_i(y_i - \hat{y}_i)
\]  

(21)

Let us not the the observation error:

\[
\hat{u}_i = u_i - \hat{u}_i
\]  

(22)

The observation error dynamics is then obtained from equation (19) and (21).

\[
\dot{\hat{u}}_i = g(\hat{u}_i, \zeta_{1i}) + \lambda_i(\zeta_{1i})
\]

The convergence is proved by the following Lyapunov candidate function:

\[
V_i = \frac{1}{2}\hat{u}_i^2
\]  

(23)

The time derivative of \( V \) is then:

\[
\dot{V}_i = \dot{\hat{u}}_i\hat{u}_i
\]  

(24)

from 20, we obtain:

\[
\dot{V}_i = \hat{u}_i \left[ \frac{1}{b_{ii}}(-a_{ii}u_i + \zeta_{1i}) - \lambda_i(\zeta_{1i}) \right]
\]  

(25)

and then as \( \zeta_{1i} \) is measured or reconstructed by a observer we choose \( \lambda_i = \frac{1}{b_{ii}} \), on \( \dot{V}_i < 0 \),

\section{IV. EXPERIMENTAL RESULTS}

In this section, we present some experimental results to validate our approach. Several trials have been done with a vehicle (Peugeot 406 of LCPC) equipped with different sensors.

Some tests were carried out at the Road and Bridges Centaral Laboratory (LCPC) test track with an instrumented vehicle. Measures have been acquired with the vehicle rolling at several speeds.

The signal measured by a Longitudinal Profile Analyser (LPA) constitutes in this experiment our reference profile.

The figure 3 shows the longitudinal vehicle speed variations.

![Fig. 3. Longitudinal velocity of the vehicle](image)

The Figure 4 shows clearly that the estimated displacements of the four wheels converge quickly to the measured ones. The curves are superposed.

The figure 5, presents the estimation errors for displacements of the wheels. The estimation error stay less than
In the figure 6, we present the roll angle and the pitch angles.

A good reconstruction of state enables the estimation of the unknown inputs of the system. Figure 7 presents both the measured road profile (coming from the LPA instrument) and the estimated one. We can observe that the estimated values are quite close the measured ones.

V. CONCLUSION

In this paper, we present enhancement of previously proposed method to estimate the road profile elevation based on second-order sliding-mode. The gains of the proposed observer are chosen very easily ignoring the system parameters. This observer is compared, using experimental data. This observer is better than the previous one in convergence and do not assume that velocity is constant. This is due to robustness of the second order Sliding mode observer which allows better rejection of perturbation and then a better reconstruction of the unknown inputs. The latter reconstruction has been also enhanced.

The estimation scheme build up using a Second Order Sliding Mode observers has been tested on experimental data (acquired with a P406 vehicle) and shown to be very efficient. The experimental results prove effectiveness and robustness of the proposed method. In our further investigations the estimations produced on line will be used to define a predictive control to enhance the safety.

REFERENCES


[14] A. Pisano, and E. Usai, "Output-feedback control of an under-
Fig. 6. Estimation of the roll and the pich angles

Fig. 7. Comparison between observers approach and LPA profile