Semi passive walking of a 7 DOF biped robot
N.Khraief & N.K.M’Sirdi
khraief@lrv.uvsq.fr, nkms@free.fr
Laboratoire de Robotique de Versailles
Université de Versailles Saint Quentin en Yvelines
10, Avenue de l’Europe 78140 Vélizy

ABSTRACT

This paper is concerned with the passive walking of an underactuated biped robot. First, we present the modelling of the biped robot (that is a 7 DOF robot). Then, we focus on the study of the almost passive dynamic walking. In particular, we show that the application of a nonlinear feedback control to stabilize the torso and knees leads the biped robot to perform a stable almost-passive dynamic walking when dealing with motions on a downhill a slope. More such control also leads the system trajectories to converge towards stable limit cycles. In this context, we present some results based on both Poincaré map method and trajectory sensitivity analysis to efficiently characterize the stability of the almost-passive limit cycles.

1 INTRODUCTION

Biped locomotion is one of the most sophisticated forms of legged locomotion. A biped robot is a two-legged robot, which may have knees and it most closely resembles human locomotion, which is interesting to study and is very complex. There has been extensive research over the past decade to find mechanical models and control systems which can generate stable and efficient biped gaits.

This paper deals with the use of stabilization of periodic gaits for control of planar biped robots. Indeed, we investigate the existence and the control of passive limit cycles.

The notion of obtaining passive gaits from mechanical models was pioneered by McGeer [5]. These gaits are only powered by gravity and are thus very energy efficient. In passive human walking, a great part of the swing phase is passive, i.e. no muscular actuation is used. Activation is primarily during the double contact phase and then it essentially turns off and allows the swing leg to go through like a double pendulum, McMahon [4]. Passive walking has been studied by several researchers, Collins, Garcia, Goswami [6], McGeer [5], M.W.Spong [8]. These have found stable passive gaits for shallow slopes where gravity powers the walk down the slope. However, it is found that no stable passive gaits exist for level ground. Consequently, a complementary control schemes is required. Thus, we will present some theoretical and simulation results based on the use of a recent control method (referred to as Controlled Limit Cycle [1] [2] [3]), which considers the system energy for both controller design and system stabilization.

Motivated by the work done so far [14] [15] [22] [23] and potential applications of biped locomotion, we conduct this paper on study of energy based control of biped robots. A new method is used to stabilize periodic cycles for such systems. The control objective is
regulation of the system energy (using a nominal energetic representation) for stabilization of
the walking robot with different speeds. The CLC control allows to establish this task.

We found that it is possible to develop energy mimicking control laws which can
make the biped gaits stable and robust under various kind of environment. They can also be
very energy efficient and make the biped gait anthropomorphic.

The paper will be organized as follows:

First we will present the modelling of the biped robot under consideration (that is a
kneed robot with torso with 7 DOF). Then, we will focus on the study of the passive dynamic
walking of this robot, on inclined slopes.

Moreover, we will show that under a simple control applied to stabilize a posture,
trajectories of the biped robot can converge towards stable limit cycles. In this context, we
will present some results based on Poincaré map method and trajectory sensitivity analysis to
efficiently characterize the stability of the almost-passive limit cycles.

However, as such limit cycles may not exist for all ground configurations, a
complementary control schemes is required. Thus, we will present some theoretical and
simulation results based on the use of a recent control method (referred to as Controlled Limit
Cycle [1] [2] [3]), which considers the system energy for both controller design and system
stabilization. A very interesting idea is to use the torso and the slope to control and stabilize
the robot at a desired speed.

Finally, some potential extensions for future works will be discussed.

2 THE BIPED ROBOT MODEL

The dynamic model of a simple planar biped robot is considered in this section. It’s shown
in figure (1). The robot has five degrees of freedom. It consists of a torso, two rigid legs, with
no ankles and two knees, connected by a frictionless hinge at the hip. This linked mechanism
moves on a rigid ramp of slope \( \gamma \). During locomotion, when the swing leg contacts the ground
(ramp surface) at heel strike, it has a plastic (no slip, no bounce) collision and its velocity
jumps to zero. The motion of the model is governed by the laws of classical rigid body
mechanics. It’s assumed that walking cycle takes place in the sagittal plane and the different
phases of walking consist of successive phases of single support. With respect to this
assumption the dynamic model of the biped robot consists of two parts: the differential
equations describing the dynamic of the robot during the swing phase, and the algebraic
equations for the impact (the contact with the ground).

2.1 Swing phase model

During the swing phase the robot is described by differential equations written in the
state space as follows:

\[
\dot{x} = f(x) + g(x)u
\]  

(1)
Where $x = (q, \dot{q})$.

(1) is derived from the dynamic equation between successive impacts given by:

$$M(q) \ddot{q} + H(q, \dot{q}) = Bu$$

Where $q = (q_1, q_{31}, q_{32}, q_{41}, q_{42})$, $u = (u_1, u_2, u_3, u_4)$: $u_1$ and $u_2$ are the torques applied between the torso and the stance leg, and the torso and the swing leg, respectively, $u_3$ and $u_4$ are torques applied at both knees $M(q) = [5 \times 5]$ is the inertia matrix and $H(q, \dot{q}) = [5 \times 1]$ is the coriolis and gravity term (i.e.: $H(q) = C(q, \dot{q}) + G(q)$) while $B$ is a constant matrix. The matrices $M, C, G, B$ are developed in [17].

2.2 Impact model

The impact between the swing leg and the ground (ramp surface) is modelled as a contact between two rigid bodies. The model used here is from [18], which is detailed by Grizzle & al. in [16]. The collision occurs when the following geometric condition is met:

$$x_2 = z_2 \text{tg} \gamma$$

Where:

$$x_2 = L_2 (\sin q_{31} - \sin q_{32}) + L_4 (\sin q_{41} - \sin q_{42})$$

$$z_2 = L_2 (\cos q_{32} - \cos q_{31}) + L_4 (\cos q_{42} - \cos q_{41})$$

Yet, from biped’s behaviour, there is a sudden exchange in the role of the swing and stance side members. The overall effect of the impact and switching can be written as:

$$h : S \rightarrow \chi$$

$$x' = h(x^-)$$

Where: $S = \{(q, \dot{q}) \in \chi / x_2 - z_2 \text{tg} \gamma = 0\}$, with $h$ is specified in [ ].The superscripts (-) and (+) denote quantities immediately before and after impact, respectively.

2.3 Overall model

The overall 7-dof biped robot has as hybrid model:

$$\dot{x} = f(x) + g(x)u$$

$$x^- (t) \notin S$$

$$x^+ = h(x^-)$$

$$x^- (t) \in S$$

This can be written

$$\dot{x} = f(x, \Lambda) u + g(x, \Lambda), \ \forall x(t) \notin C(e_i)$$

$$x^+ = h(x^-, e_i), \ \forall x(t) \in C(e_i)$$

Where the discrete state set is $\Lambda = \{s_1, s_2\} = \{\Lambda_1, \Lambda_2\}$. $C(e_i)$ is state change condition event after impact for the event $e_i$. The states are $s_1$ (leg n°1 is the support leg) and $s_2$ (leg n°2 is the support leg). The two events correspond to single supports on one leg and then the other. The commutation events or changes are in the set $E = \{e_1, e_2\} = \{(s_1, s_2), (s_2, s_1)\}$. We can consider now this model and transform the system by a first partial feedback, in order to allow existence of Limit Cycles corresponding to walking downhill a slope.

3 SEMI PASSIVE DYNAMIC WALKING

3.1 Outline of procedure

Our objective is to examine the possibility that a kneed biped robot with torso can exhibit a passive dynamic walking in a stable gait cycle, downhill a slope. In our previous work [14]
on a kneeless biped robot with torso, we have shown that such systems can steer a passive dynamic walking on inclined slopes, one time they have been transformed by a partial feedback control. Thus, the main idea of this work is to transform the 7-DOF biped robot in a system for which there exists limit cycles. The objective is then, to obtain nearly passive Limit Cycles, which will be exploited further to get a dynamic walking behaviour. This is possible when a torque is applied to stabilize the torso and knees at fixed positions. A partial feedback will be designed for, in the next section.

3.2 Non collocated input/output linearization

In this section we use some results from [9] [10] [11] [24], the objective is to get a control scheme able to stabilize the torso at a desired position.

To show this we may write the dynamic equations system (2) as:

\[
\begin{align*}
M_{11}\ddot{q}_1 + M_{12}\ddot{q}_2 + h_1 + \phi_1 &= \tau_1 \\
M_{21}\ddot{q}_1 + M_{22}\ddot{q}_2 + h_2 + \phi_2 &= \tau_2
\end{align*}
\]  

(9)

(10)

Where \( \theta_1 = q_t \) and \( \theta_2 = (q_{31}, q_{32}, q_{41}, q_{42})^T \) and:

\[
M(q) = \begin{bmatrix} M_{11}(q) & M_{12}(q) \\ M_{21}(q) & M_{22}(q) \end{bmatrix}
\]

(11)

is the symmetric, positive definite inertia matrix, the vector functions \( h_1(q, \phi) \in \mathbb{R}^1 \) and \( h_2(q, \phi) \in \mathbb{R}^4 \) contain coriolis and centrifugal terms, the vector functions \( \phi_1(q) \in \mathbb{R}^1 \) and \( \phi_2(q) \in \mathbb{R}^4 \) contain gravitational terms and \((\tau_1, \tau_2)^T = Bu \) represents the input generalized force produced by the actuators, such that: \( \tau_1 = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} u \), et \( \tau_2 = (-1, -1, 0, 0) \).

From (9)-(10) we obtain:

\[
\begin{align*}
M_{21}\ddot{q}_1, h_2 + \phi_2 - \tau_2 &= M_{22}v_2 \\
\ddot{v}_2 &= v_2
\end{align*}
\]  

(12)

(13)

In order to steer some trajectories for the knees, we have to partition the vector \( v_2 \) in two parts from (12-13) we obtain the following expression:

\[
\begin{bmatrix}
M_{21}\ddot{q}_1, h_2 + \phi_2 - \tau_2 \\
\ddot{v}_2
\end{bmatrix}
= 
\begin{bmatrix}
\ddot{v}_1 \\
\ddot{v}_2
\end{bmatrix}
= 
(M_{21}; M_{22})
\]

(14)

\[
\begin{bmatrix}
\ddot{v}_1 \\
\ddot{v}_2
\end{bmatrix}
= 
(v_{21}, v_{22}) \ ; 
M_{22} = [M_{21}; M_{22}]
\]

(15)

As \( \text{rank}(M_{22}(q)) = 1 \) for all \( q \in \mathbb{R}^4 \), the system (9)-(10) is said to be strongly inertially coupled. Under this assumption we may compute a pseudo-inverse matrix \( M_{12} = M_{12}^T(M_{12}M_{12}^T)^{-1} \) and define \( v_{21} \) in (13):

\[
v_{21} = -M_{21}^{-T}(M_{21}v_1 + h_2 + \phi_2 - \tau_2 + M_{222}v_{22})
\]

(16)

Or \( v_1 \in \mathbb{R}^1 \), et \( v_{22} \in \mathbb{R}^2 \) are additional controls chosen as follows:

\[
\begin{align*}
v_1 &= \ddot{q}_1 + k_{d1}(\ddot{q}_1 - \ddot{q}_1) + k_{p1}(q_1 - q_1) \\
v_{22} &= \begin{bmatrix}
\ddot{q}_{22} + k_{d1}(\ddot{q}_{22} - \ddot{q}_{22}) + k_{p1}(q_{d2} - q_{d2}) \\
\ddot{q}_{23} + k_{d1}(\ddot{q}_{23} - \ddot{q}_{23}) + k_{p1}(q_{d3} - q_{d3}) + k_{p2}(q_{d4} - q_{d4}) \\
\ddot{q}_{24} + k_{d1}(\ddot{q}_{24} - \ddot{q}_{24}) + k_{p1}(q_{d5} - q_{d5}) + k_{p2}(q_{d6} - q_{d6}) + k_{p3}(q_{d7} - q_{d7})
\end{bmatrix}
\end{align*}
\]

(17)
Note also that we can choose a desired trajectory to avoid hitting the ground, for example such as:

\[ q_4^d = F_a \sin \left[ \frac{\pi}{t_{\text{max}}} t \right] \], \quad q_{31}^d = q_{31} - \epsilon \quad \text{and} \quad q_1^d = Cte

The actual control \( u \) is given by combining (14), (16) and (17), after some algebra as:

\[ u = Mz_1v_1 + Nv_2 + \phi_0 + \phi_0^0 \quad (18) \]

The non collocated linearization approach transfers the system (2) into two subsystems written as follows:

\[ \dot{\eta} = A\eta \]
\[ \ddot{\xi} = s(\eta, z, t) \quad (19) \]

Where \( \eta = (\eta_1, \eta_2)^T = (q_1 - q_1^d, \dot{q}_0 - \dot{\phi}_0^0)^T \), \( z = (z_1, z_2)^T = (q_2, \dot{q}_2)^T \), \( A = \begin{pmatrix} 0 & 1 \\ -k_p & -k_d \end{pmatrix} \) and

\[ s(\eta, z, t) = \begin{pmatrix} z_2 \\ -M_{12}(h_1 + \phi_1) - M_{11}(\phi_2 - k_p \eta_1 - k_d \eta_2) \end{pmatrix} \quad (20) \]

We refer to the linear subsystem as \( \eta \)-subsystem. Accordingly the non linear subsystem in (19) is denoted as \( z \)-subsystem where \( s(0, z, t) \) defines the zero dynamics relative to the output \( \eta \).

3.3 Finding period one gait cycles and step period

After stabilizing the torso and the knees we proceed to simulate the motion of the biped robot downhill a slope. The walker’s motion can exhibit periodic behaviour. Limit cycles are often created in this way. At the start of each step we need to specify initial conditions \( (q, \phi) \) such that after \( T \) seconds (\( T \) is the minimum period of the limit cycle) the system returns to the same initial conditions at the start. A general procedure to study the biped robot model is based on interpreting a step as a Poincaré map. Limit cycles are fixed points of this function. A Poincaré map samples the flow \( \phi \) of a periodic system once every period [21]. The concept is illustrated in figure (2). The limit cycle \( \Gamma \) is stable if oscillations approach the limit cycle over time. The samples provided by the corresponding Poincaré map approach a fixed point \( x^* \). A non stable limit cycle results in divergent oscillations, for such a case the samples of the Poincaré map diverge.

![Figure (2) Poincaré map](image)

3.4 Gait cycle stability

Stability of the Poincaré map (20) is determined by linearizing \( P \) around the fixed point \( x^* \), leading a discrete evolution equation:

\[ \Delta x_{k+1} = DP(x^*) \Delta x_k \quad (21) \]

\(^1\) The matrix \( A \) must be Hurwitz.
The major issue is how to obtain $DP(x)$ - The Jacobean matrix- while the biped dynamics is rather complicated; a closed form solution for the linearized map is difficult to obtain. But one can be obtained by the use of a recent generalization of trajectory sensitivity analysis [20] [21] [14] [15].

### 3.5 Numerical procedure

A numerical procedure [20] [21] [14] [15] is used to test the walking cycle via the Poincaré map, it’s resumed as follows:

1. With an initial guess we use the multidimensional Newton-Raphson method to determine the fixed point $x^*$ of $P^*$ (immediately prior the switching event).
2. Based on this choice of $x^*$, we evaluate the eigenvalues of the Poincaré map after one period by the use of the trajectory sensitivity.

![Diagram](image)

Figure (3) Algorithm for the numerical analysis

### 3.6 Simulation results

Consider the model (1), with the following values:

<table>
<thead>
<tr>
<th>Paramètres</th>
<th>Tronc</th>
<th>Fémur</th>
<th>Tibia</th>
</tr>
</thead>
<tbody>
<tr>
<td>Masse (kg)</td>
<td>$M_T$  = 58</td>
<td>$M_1 = 6.8$</td>
<td>$M_4 = 3.2$</td>
</tr>
<tr>
<td>Longueurs (m)</td>
<td>$L_T = 0.5$</td>
<td>$L_1 = 0.4$</td>
<td>$L_4 = 0.4$</td>
</tr>
<tr>
<td>Position des centres de masses (m)</td>
<td>$Z_T = 0.2$</td>
<td>$Z_1 = 0.16$</td>
<td>$Z_4 = 0.128$</td>
</tr>
<tr>
<td>Premiers moments d’inerties (kg m²)</td>
<td>$MY_1 = 0.01$</td>
<td>$MZ_1 = 4$</td>
<td>$MZ_3 = 1.11$</td>
</tr>
<tr>
<td></td>
<td>$MZ_4 = 0.41$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Moments d’inerties d’ordre 2 (kg m²)</td>
<td>$XX_1 = 2.22$</td>
<td>$XX_3 = 1.08$</td>
<td>$XX_4 = 0.93$</td>
</tr>
</tbody>
</table>
We choose the hyper plane $\Sigma$ as the event plane. We let the biped robot on a downhill slope with the control scheme (18). Starting with a suitable initial guess we obtain the following result:

$$\gamma = 0.075 \text{rad}, q^d = -0.1 \text{rad}, K_{p_1} = 400, K_{v_1} = 120, K_{p_2} = 500, K_{v_2} = 100, K_{p_3} = 500, K_{v_3} = 100$$

we obtain

$$x = [3.188436; 3.042832; 2.857767; 2.825776; -0.100000; -0.838264; 0.691926; 0.818408; 0.441655; -1.149617]$$

5 CONCLUSION

In this paper we present that by the use of a simple nonlinear feedback control with a numerical optimization, we can obtain a semi passive walking gait. The gaits correspond to limit cycles or region of attraction defined with the energy of the almost passive walking on the downhill slope. The approach is applied to a 7 DOF biped robot. Simulation results emphasize performance and efficiency of the proposed methodology.

REFERENCES:
[18] Hurmuzlu, Y., and Marghilu, D.B, Rigid body collisions of palanar kinematic chains with multiple contact points, the international journal of robotics research, 13(1) : 82-92, 1994