Nonlinear longitudinal tire force estimation based sliding mode observer

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Abstract—This paper presents an estimation method for vehicle longitudinal dynamics, particularly the tractive/braking force. This estimation can be used to detect a critical driving situation to improve the security. It can be used also in several vehicle control systems such as Anti-lock brake systems (ABS), traction control system (TCS), diagnostic systems, etc... The main characteristics of the vehicle longitudinal dynamics were taken in to account in the developed model used to design observer and computer simulations. The state variables of system are the angular wheel velocity, vehicle velocity and the longitudinal tire force. The proposed differential equation of the tractive/braking force is derived from the concept of relaxation length. The observer designed is based on the sliding mode approach by using only the angular wheel velocity as measurement. The proposed method of estimation is verified through one-wheel simulation model with a “Magic formula” tire model. Simulations results show an excellence reconstruction of the tire force.

Keywords—Nonlinear observer, state estimation, sliding mode, tire forces, wheelslip, relaxation length.

I. INTRODUCTION

The active safety is becoming important due to recent research on intelligent transportation systems (ITS) technology. Several safety systems has been developed for cars (ABS, TCS the collision warning/avoidance control system, etc.). To increase the safety it will be more interesting to determine the accident causes (driver error, dysfunctionnement of the transportation system, etc.). The reconstruction of the accident scenario is a mean to determine the accident causes. The reliability of this reconstruction require a pertinent description of the tire/ground interface characteristics as was proved in [1]. The tire forces properties influence on vehicle dynamic performance [2]. Thus it is necessary to take into account this characteristics. However, tire forces and road friction (μ) are difficult to measure directly. Their values are often deduced by some dynamic tire models. The tire models performed in literature are complex and depend on several factors (load, tire pressure, environmental characteristics, etc.), (see references [3],[4], [5], [6], [7], [8]). This models has been developed particularly for vehicle dynamic simulations. They depend on several parameters and they are generally derived from experimental data based machine tests. The basic parametric model which is often used to describe the tire/road characteristics is the so-called “Magic formula” (see references [5], [8]).

To dispose on relevant information on tire forces which are necessary in development of vehicle control systems, to alarm the driver for a sudden changes, etc..., we have generally recourse to the estimation methods, thus many hard-to-measure signals have to be observed or estimated. Recently, many studies on the tire/road characteristics estimation have been performed. Ray [9] estimates the tire forces with an extended Kalman filter. The method of estimation use an eight degree of freedom vehicle model without requires tire forces model. The forces are considered as unknown parameters. A random walk model is added to determine the unknown forces and a Bayesian approach is used to estimate μ. Gustafsson in [10] presented a tire/road friction estimation method based Kalman filter to give a relevant estimation of the slope of the curve μ versus slip (s). Slip is a variable that depends on the relative difference in wheel velocities. Yi et al [11] proposed for estimation of tire road/friction two methods: an observer-based least square method and an observer/filtered regressor based method. These methods were designed by using an empirical tire model with nominal value of the tire/road friction which will be actualized by observer. Canudas et al [12] proposed a method for observation of the tire/road characteristics by estimating on-line the changes on road condition. This method is based on dynamic tire/road model proposed in [13].

The work presented here is a part of project “wet road accident” of PREDr program. The goal of this part is to evaluate a relevant information regarding tire/ground forces based upon a simplified model from which system state will be estimated. In this paper we deals only the pure longitudinal dynamic of vehicle and we develop a method for longitudinal tire force estimation. The method is based on the sliding mode approach [14], [15] known to be robust to parametric uncertainties and perturbations. A simplified longitudinal vehicle model is developed in this study to design observer and for computer simulations. The state variables considered in this model are angular wheel velocity, vehicle velocity and tractive/braking force. The equations of the two first states are determined by Newton’s
law. The last state is derived from the relaxation length concept [16], [17], [18]. A first order slip model based relaxation length concept is used to establish a differential equation of the ttractive/braking force.

In the next section, we describe the system dynamics and gives a state form of system. The system observability is study in section III. In section IV a design of the sliding mode observer is proposed, and observer stability analysis and observation error convergence conditions are presented. Simulation results are given in section V. Conclusion and perspective of this work are presented in section VI.

II. SYSTEM DYNAMICS

In this section, we describe the model for vehicle longitudinal dynamic. This model will then be used for system analysis, observer design and computer simulations. The model describe in this study retains the main characteristics of the longitudinal dynamic. The state variables of this system are: the angular wheel velocity, vehicle velocity, and longitudinal tire force. The input signal is the torque applied to the wheel. The application of Newton’s law to wheel and vehicle dynamics gives the two first state equations. The last state equation is derived from the concept of the relaxation length.

The dynamic equation for the angular motion of the wheel is

\[ J\ddot{\omega} = T - f_w\omega - rF_t \]  

where \( J \) is the moment of inertia of the wheel, \( f_w \) is the viscous rotational friction and \( r \) is the radius of the wheel. The applied torque \( T \) result from the difference between the shaft torque from the engine and the break torque. \( F \) is the tire tractive/braking force which result from the deformation of the tire at the tire/ground contact patch.

The vehicle motion is governed by the following equation

\[ m\ddot{v} = F - c_xv^2 \]  

where \( c_x \) is the aerodynamic drag coefficient and \( m \) is the vehicle mass.

The tractive (braking) force, produced at the tire/road interface when a driving (braking) torque is applied to a pneumatic tire, oppose the direction of relative motion between the tire and road surface. This relative motion determines the tire slip properties. The slip is due to deflection in the contact patch ([2], [6]). Thus longitudinal slip \( (s) \) is associated with the development of tractive or braking force and we can express \( F \) as:

\[ F = f(s) \]  

The longitudinal wheelslip is generally called the slip ratio and it is describe by a kinematic relationship as:\(^1\):

\[ \begin{cases} \dot{s} = \frac{\omega}{r} & \text{during braking phase} \\ \dot{s} = \frac{\omega}{r} & \text{during traction phase} \end{cases} \]  

\(^1\)the upper bar is introduced here to denote the steady-state of slip with respect to transient slip

where \( v_s = v - r\omega \) represent the slip velocity in the contact patch.

Recently many advanced researchers ([19], [20]) have studies the behavior of the tire properties in rapid transient maneuvers such cornering on uneven roads, brake torque variation and oscillatory steering. This studies deals with transients in tire force and use the concept of the relaxation length to account the deformation of the carcass in the contact patch that is responsible for the lag in the response to lateral and longitudinal slip. The motivation for this studies is to improve understanding of the tire behavior with respect to experimental results and then include it in vehicle dynamic simulations. The concept of relaxation length has been formulated particularly for the lateral dynamic to model transient tire behavior (see references: [21], [22], [17]), thus this concept has been adapted for longitudinal dynamic. In [23], [24] the authors have used the relaxation length concept to describe the longitudinal and lateral forces to studies the tire dynamic behavior which is represented by a rigid ring model. In [16], [18] the authors gives a review to the concept of relaxation length and presents a formulation for both the longitudinal slip and slip angle as state variables which well be used with any semi-empirical tire model. The pure longitudinal slip can be presented by a first order relaxation length as:

\[ \begin{cases} \dot{s} + vs = v_s & \text{during braking} \\ \dot{s} + r\omega s = v_s & \text{during traction} \end{cases} \]  

The steady-state solution of equations (5) equals the normal definition of longitudinal slip given by (4). It is found that the relaxation length equals the slip stiffness \( (C_s) \) divided by the longitudinal stiffness in the contact patch \( (\kappa_s = 2k_s l) \):

\[ \sigma = \frac{C_s}{\kappa_s} \]  

where \( l \) is the half contact length and \( k_s \) is the stiffness per unit of length of the tread.

The slip stiffness is defined as the local derivative of the stationary tire force-slip characteristic (see Fig. 1):

\[ C_s = \frac{\partial F}{\partial s} \]  

According to equations (3) and (7) we can suggest a formulation of the derivative of \( F \) as:

\[ \dot{F} = C_s \dot{s} \]  

During braking phase, the equation (8) become:

\[ \sigma \dot{F} = C_s (vs + v_s) \]  

However at small slip, we can write:

\[ F = C_s \frac{\partial F}{\partial s} \bigg|_{s=0} = 2k_s l^2 \]
Then equation (9) can be write as:

$$\sigma F = -vF + C_s v_s$$ (12)

Note that for small slip, the obtained equation (11) is like the one in ([25]) used to generate longitudinal force with a rigid ring model. Without much details, we can extended the equation (12) to large slip and we propose a derivative of $F$ as:

$$\sigma \dot{F} = -v(\dot{F} - F_0) + C_s v_s$$ (13)

the unknown parameter $^2$ $F_0$ is the intersection of the slop $\frac{\partial F}{\partial s}$ and the F-axis (see Fig. 1).

Finally, the model can be presented during braking in the state-variable form by choosing the following state variables:

$$x_1 = \omega, \quad x_2 = v, \quad x_3 = F$$

and by defining the unknown parameters as:

$$\theta_1 = \frac{F_0}{\sigma}, \quad \theta_2 = \frac{1}{\sigma}$$

Then equation (9) can be write as:

$$\begin{align*}
\frac{dx}{dt} &= f(x) + gu \\
y &= h(x)
\end{align*}$$ (14)

with

$$f(x) = \begin{bmatrix}
-\frac{F}{m} x_1 - \frac{v}{m} x_3 \\
-\frac{F}{m} x_2 + \frac{v}{m} x_3 \\
\kappa_s x_1 - \kappa_x x_2 + \theta_1 x_2 - \theta_2 x_2 x_3
\end{bmatrix}$$

$$g = \begin{bmatrix}
\frac{1}{\sigma} \\
0 \\
0
\end{bmatrix}, \quad h(x) = \begin{bmatrix}
x_1 \\
0 \\
0
\end{bmatrix}, \quad u = T$$

### III. System Observability

To prove the observability of the system given in (14), we use the observability rank criterion ([26]) based on the Lie derivative, (this concept gives a local observability). The Lie derivative of the output $h$ along the field vectors $f$ is given by:

$$L_f h(x) = \frac{\partial h}{\partial x} f(x)$$ (15)

The construction of the observability matrix is given by repeated Lie derivative as follows:

$$O(x) = \begin{bmatrix}
dh \\
d(L_f h) \\
\vdots \\
d^{n-1}(L_f h)
\end{bmatrix}$$ (16)

where $n$ is the dimension of the state space of the system.

For system (14), we obtain the following observability matrix:

$$O(x) = \begin{bmatrix}
1 \\
-\frac{F}{m} x_2 + f^2 \\
\frac{r(K_s - \theta_1 + \theta_2 x_3)}{j^2} \\
\frac{r(f_j + f_2 x_2)}{j^2}
\end{bmatrix}$$ (17)

The observability matrix is full rank for any states, then system (14) is locally observable everywhere.

### IV. Nonlinear Observer Design

From (14) we can see that nonlinear system is linear in the unknown parameters and system (14) can be rewritten as:

$$\begin{align*}
\dot{x} &= \xi(x) + \Psi(x) \theta + gu \\
y &= h(x)
\end{align*}$$ (18)

with

$$\xi(x) = \begin{bmatrix}
-\frac{F}{m} x_1 - \frac{v}{m} x_3 \\
-\frac{F}{m} x_2 + \frac{v}{m} x_3 \\
\kappa_s x_1 - \kappa_x x_2
\end{bmatrix}$$

$$\Psi(x) = \begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
x_2 - x_2 x_3
\end{bmatrix}, \quad \theta = \begin{bmatrix}
\theta_1 \\
\theta_2
\end{bmatrix}$$

Then the unknown parameters can be estimated. To estimate both state and unknown parameters we propose the following observer based sliding mode:

$$\begin{align*}
\dot{x} &= \hat{\xi} + \Psi(\hat{x}) \hat{\theta} + gu + \Lambda_s \\
\dot{\hat{\theta}} &= \eta
\end{align*}$$ (19)
with
\[
\dot{\xi} = \begin{bmatrix}
-\frac{\nu}{m} g - \frac{\lambda_0}{m} \dot{x}_3 \\
-\frac{\nu}{m} \dot{x}_2 + \frac{\lambda_1}{m} \dot{x}_3 \\
\kappa_1 r y - \kappa_2 \ddot{x}_2
\end{bmatrix}
\]
\[
\dot{\lambda}_s = \begin{bmatrix}
\lambda_1 \\
\lambda_2 \\
\lambda_3
\end{bmatrix}
\cdot \text{sign}(x_1 - \dot{x}_1), \quad \Psi(\dot{x}) = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}
\begin{bmatrix}
\ddot{x}_2 \\
\ddot{x}_3
\end{bmatrix}
\]
\[
\dot{\theta} = \begin{bmatrix}
\dot{\theta}_1 \\
\dot{\theta}_2
\end{bmatrix}, \quad \eta = \begin{bmatrix}
\eta_1 \\
\eta_2
\end{bmatrix}
\]
where \( \dot{x}^T = (\dot{x}_1, \dot{x}_2, \dot{x}_3) \) represents the estimated state variables and \( (\dot{\theta}_1, \dot{\theta}_1) \) are the estimated of the unknown parameters. The observer gains \( \lambda_1 \) and variables \( (\eta_1, \eta_2) \) will be defined thereafter. The single measurement is \( x_1 \).

For the observer stability study and the observation error convergence conditions, we define the following observation errors:
\[
\begin{align*}
\bar{x}_1 &= x_1 - \dot{x}_1 \\
\bar{x}_2 &= x_2 - \dot{x}_2 \\
\bar{x}_3 &= x_3 - \dot{x}_3 \\
\bar{\theta}_1 &= \theta_1 - \dot{\theta}_1 \\
\bar{\theta}_2 &= \theta_2 - \dot{\theta}_2
\end{align*}
\]
Then, we have the error equation:
\[
\begin{cases}
\dot{\bar{x}}_1 = \bar{\xi} + \Psi(x) \theta - \Psi(\dot{x}) \dot{\theta} - \Lambda_s \\
\dot{\bar{\theta}} = -\eta
\end{cases}
\]
with
\[
\bar{\xi} = \begin{bmatrix}
-\frac{\nu}{m} \bar{x}_3 \\
-\frac{\nu}{m} (\bar{x}_2 - \bar{x}_2^3) + \frac{\lambda_0}{m} \bar{x}_3 \\
-\kappa_2 \bar{x}_2
\end{bmatrix}
\]
and where
\[
\begin{align*}
\Psi(x) \theta &- \Psi(\dot{x}) \dot{\theta} = \Psi(\bar{x}) \dot{\theta} + (\Psi(x) - \Psi(\dot{x})) \dot{\theta} \\
x_2^3 - \bar{x}_2^3 &= 2\bar{x}_2 \ddot{x}_2 + \ddot{x}_2^2
\end{align*}
\]
Now, consider the sliding surface:
\[
S = \{ \bar{x}_1 = 0 \}
\]
The sliding condition that guarantees the attractivity of \( S \) is given by:
\[
\dot{\bar{x}}_1 \dot{\bar{x}}_1 < 0
\]
which can be rewritten as:
\[
\bar{x}_1 (-\frac{r}{J} \bar{x}_3 - \lambda_1 \text{sign}(\bar{x}_1)) < 0
\]
The inequality (25) is verified if \( \lambda_1 \) is chosen such that
\[
\lambda_1 > \frac{r}{J}\bar{x}_3|_{\max}
\]
with \( |\bar{x}_3|_{\max} \) is the maximum value of \( \bar{x}_3 \) at any time, then \( \bar{x}_1 \) converges to \( x_1 \) in finite time and stay equal to the actual value for all \( t > t_0 \).

Moreover, we have also:
\[
\dot{x}_1 = 0 \quad \forall t > t_0
\]
This one provides an equivalent form of the “sign” on sliding surface and we have
\[
\text{sign}(\bar{x}_1) = -\frac{r}{J\lambda_1} \bar{x}_3
\]
By substituting the equivalent form of “sign” in remained equations of the system (20), by developing and using equations ((21) and (22)), we obtain the following reduced sliding dynamics:
\[
\begin{cases}
\dot{\bar{\xi}} = -\chi(\bar{\xi}) \bar{\xi} + \Psi(\bar{\xi}) \dot{\bar{\theta}} - \Theta \bar{\xi} \\
\dot{\bar{\theta}} = -\eta
\end{cases}
\]
with
\[
\chi(\bar{\xi}) = \begin{bmatrix}
\frac{\nu}{m} \bar{x}_3^2 \\
-\frac{\nu}{m} \bar{x}_2^2 + \frac{\lambda_0}{m} \bar{x}_3 \\
-\kappa_2 \bar{x}_2
\end{bmatrix}
\]
\[
\Theta = \begin{bmatrix}
\theta_2 \bar{x}_3 - \theta_1 & 0 \\
\theta_2 (\ddot{x}_2 + \ddot{x}_2) & \bar{x}_2
\end{bmatrix}
\]
where \( \xi^* = \begin{bmatrix} x_2^3 \\
x_3^2 \\
x_3 \end{bmatrix} \)

To show that the remained state variables and the unknown parameters converge toward zero, we consider the following Lyapunov function:
\[
V(\bar{\xi}, \bar{\theta}) = \frac{1}{2} \bar{\xi}^T \bar{\xi} + \frac{1}{2} \bar{\theta}^T Q^{-1} \bar{\theta}
\]
where \( Q = \text{diag}(q_1, q_2) \) is a positive diagonal matrix, then
\[
\dot{V} = \bar{\xi}^T \dot{\bar{\xi}} + \dot{\bar{\theta}}^T P^{-1} \dot{\bar{\theta}}
\]
\[
\dot{\bar{\xi}} = -\chi(\bar{\xi}) \bar{\xi} + \bar{\xi}^T \Psi(\bar{\xi}) \dot{\bar{\theta}} - \bar{\xi}^T \Theta \bar{\xi} - \bar{\theta}^T Q^{-1} \eta
\]
Defining the adaptive law \( \eta \) as:
\[
\eta = Q \Psi^T(\bar{\xi}) \bar{\xi}
\]
For implementation, we can develope \( \eta \) as:
\[
\eta = Q \bar{\xi}^T(\bar{\xi}) \Gamma \bar{x}_3
\]
with
\[
\Gamma = \begin{bmatrix}
1 \\
1
\end{bmatrix}
\]
Note that in sliding surface we have:
\[
\bar{x}_3 = \frac{J\lambda_1}{r} \text{sign}(\bar{x}_1)
\]
thus \( \eta \) can rewritten as:
\[
\eta = -\frac{J\lambda_1}{r} P \bar{\xi}^T(\bar{\xi}) \Gamma \text{sign}(\bar{x}_1)
\]
\( \text{sign} \) denotes the equivalent, in the mean, to the sign effect.
we obtain
\[ \dot{\mathbf{\xi}} = -\mathbf{\varepsilon}^T \mathbf{\chi}(\mathbf{\xi}) \mathbf{\varepsilon} - \mathbf{\varepsilon}^T \mathbf{\Psi} \mathbf{\xi} \] (34)
To guarantee the stability of the observer, the matrix \( \mathbf{\chi}(\mathbf{\xi}) \) must be positive definite. This one is possible by pole placement , then by choosing the gains \( \lambda_1 \) and \( \lambda_2 \) as:
\[
\begin{align*}
\lambda_2 &= J \lambda_1 \left( 4 c_x^2 \tilde{x}_2^2 - 2 m c_x \tilde{x}_2 (p_1 + p_2) \\
&\quad - m \kappa_x + m^2 \rho_2 p_2 \right) / r m^2 \kappa_x \\
\lambda_3 &= J \lambda_1 \left( 2 c_x \tilde{x}_2^2 - m (p_1 + p_2) \right) / r m
\end{align*}
\]
where \( p_1 \) and \( p_2 \) are positive poles.

Now define \( \rho_0 \) to be the minimum eigenvalue of the matrix \( \mathbf{\Psi} \):
\[
\rho_0 = \lambda_{\min}(\mathbf{\Theta}) = \min \left\{ \frac{c_x}{m} \tilde{x}_2, \theta_2 (\tilde{x}_2 + \tilde{x}_2) \right\}
\]
Then, the time derivative of \( V(\mathbf{\xi}, \mathbf{\theta}) \) can be bounded as follows:
\[
\dot{V} \leq -(p_m + \rho_0) \| \mathbf{\xi} \|^2
\]
where \( p_m, \) the minimum of \( \{p_1, p_2\}, \) is chosen such:
\[
p_m > \rho_0
\]
Consequently, \( \dot{V} \) is strictly negative and thus \( \mathbf{\xi} \) tend asymptotically to zero. However, the estimation error of the unknown parameters is only bounded.

V. Simulation results

To illustrate the performance of the proposed approach of estimation we consider a one-wheel model (equations (1) and (2)) with "Magic formula" tire model. The model parameters are listed in table (I).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m )</td>
<td>400</td>
<td>Kg</td>
</tr>
<tr>
<td>( J )</td>
<td>1.3</td>
<td>Kg ( m^2 )</td>
</tr>
<tr>
<td>( r )</td>
<td>0.3</td>
<td>( m )</td>
</tr>
<tr>
<td>( C_s )</td>
<td>0.3</td>
<td>( N/m^2s^2 )</td>
</tr>
</tbody>
</table>

TABLE I
THE PARAMETERS OF THE TYRE MODEL.

Figure (2) presents the \( \mu - \text{slip} \) characteristic used to generate the tire force to be estimated.

The equation (5) is used to calculate the value of the slip ratio for model simulation with \( \sigma = 0.09m/s \).

The total torque applied to the wheel during a braking phase is shown in figure (3). The observer gains are: \( \lambda_1 = 140, q_1 = 2.5, q_2 = 3.5 \cdot 10^{-5}, p_1 = 500 \) and \( p_2 = 300 \). We have taken an error of 25\% on the different parameter listed in table (I). The value of the longitudinal stiffness in the contact patch \( \kappa_x \) is chosen by taking the typical values for \( k_x \) and \( l \) (\( k_x = 17 \cdot 10^3 N/m^2 \) and \( l = 0.05m \) for a load \( F_z = 4000N \)). The unknown parameters \( \theta_1 \) and \( \theta_2 \) are initialized respectively -1.5 \( \cdot 10^4 N/m^{-1} \) and 15 \( m^{-1} \). The initial conditions are \( x_1(0) = y(0) = 91 rad/s \), \( x_2(0) = 30 m/s \) and \( x_3(0) = -10^4 N \). Figure (4) show the convergence of the angular wheel velocity estimation to actual value in finite time and estimation error will be zero due to the fact that angular wheel velocity is measured. In figures (5 and 6) we show respectively the asymptotic convergence of the vehicle velocity and longitudinal tire force to actual values. In simulation, the actual tire force is generated by "Magic formula" tire model.

The performance of this estimation approach is satisfactory since the estimation error is minimal for state variables. The unknown parameters can not converge necessarily to their actual values. They are be good for adaptation.

VI. Conclusion

In this paper, we have presented a new estimation method for vehicle longitudinal dynamics based sliding mode observer. The main contribution is the proposed differential equation of the tractive/braking force derived from the relaxation length concept. The interest of this method is to be able to estimate the tire force on-line by using only angular wheel velocity measurement and applied torque which can be deduced by engine RPM sensors and Throttle Position Sensor (TPS) and also the braking pressure. Simulation results was illustrated the ability of this approach to give well estimation of both vehicle velocity and longitudinal tire force. The robustness of the sliding
mode observer versus uncertainties on the model parameters has also been showed in simulation. The extended work is to applying this method to an instrumented vehicle.

References


