Wheel-Slip Regulation based on Sliding mode approach

A. EL Hadri, J.C. Cadiou, K.N. M'Sirdi
Laboratoire de Robotique de Paris, Université de Versailles

Y. Delanne
Laboratoire Central des Ponts et Chaussées de Nantes

Copyright © 2001 Society of Automotive Engineers, Inc.

ABSTRACT

This paper presents a nonlinear observer and controller based on passivity and sliding mode approach for vehicle traction control. The main contribution is the on-line estimation of the tire force which is needed for control. The concept of relaxation length describes the wheel-slip variation as a first order model. From this concept a differential equation of tire force is proposed to design a controller based on nonlinear observer. Only longitudinal dynamics are considered in this study. Stability analysis in closed-loop is proved by Lyapunov's method. Sufficient conditions for applying sliding mode based control are derived. The proposed control is verified through one-wheel simulation using "Magic formula'' tire model. The robustness of control is tested by including errors in the parameters and by changing value of tire adhesion. A comparison of the proposed approach of control with PD control is given.

INTRODUCTION

Active safety is of a growing importance due to recent research on Intelligent Transportation Systems (ITS) technology. Several safety systems has been developed for cars (ABS, TCS, Collision warning/avoidance control system, etc.). The knowledge of the characteristics of the tire/road adhesion are a crucial factors to achieve these control systems. It was shown that tire forces influence the vehicle dynamic performance [7]. However, tire forces or tire/road adhesion are difficult to measure directly. Their values are often deduced by some approximate dynamic tire models. The tire models proposed in literature are complex and depend on several factors (load, tire pressure, environmental characteristics, etc.), (see references [1,5,7,13]). These models has been developed particularly for vehicle dynamic simulations and analysis. They are generally derived from experimental data produced by machine tests.

For vehicle control systems, the knowledge of tire forces can be done through estimation methods (see references [4,9,16]), thus many hard-to-measure signals have to be observed or estimated. The vehicle traction control is realized to obtain a desired vehicle motion, maintain vehicle stability and steerability, ensure safety, etc. Many advanced researches in this field use some recent approaches of control with regard to the conventional controllers [18,19,21]. They use, for control, the cinematic relationship of wheel-slip. In order to avoid the problem of unknown tire forces they use either a nominal tire model or approximation methods.

In this paper we describe the wheel-slip variation by a first order model based on concept of relaxation length [2,6,11]. Since the wheel-slip is associated to the development of the longitudinal tire force, we propose a time derivative of tire force based on variation of wheel-slip. The proposed equation of longitudinal tire force enables on-line estimation of the unknown forces. Only pure longitudinal dynamic of vehicle is considered in this paper. A simplified longitudinal vehicle model is developed to design an observer and a control law, for stability analysis and for computer simulations. To be robust versus parametric uncertainties and perturbations, the controller and observer design are based on sliding mode approach [17,20]. The state variables considered in the dynamics model are angular wheel velocity, vehicle velocity, longitudinal wheel-slip and tractive/braking force. We consider that only angular wheel velocity is measured directly. In braking case, the braking torque can be obtained by measurement of pressure of the brakes. In acceleration case, the applied torque is obtained throughout a driven table and RMP measurement. Note that, we do not need to known exactly the value of torque for these cases.

In the next section, we describe the system dynamics. The formulation of control and a state space form for observer design are performed in section III. The system observability is studied in section VI. In section VI a design of sliding mode observer is proposed and its stability analysis is proved. In section VI, a control law based on this observer is presented. The stability analysis in closed-loop and convergence conditions of observation and control errors are shown. Simulation results are given in section VII. Finally, conclusion and perspectives of this work are presented in section VIII.
In this section, we describe the model for vehicle longitudinal dynamic. This model will then be used for system analysis, observer design and computer simulations. This model retains the main characteristics of the longitudinal dynamic. The application of Newton's law to wheel and vehicle dynamics gives the equations of motion. The concept of the relaxation length is used to deduce the differentials equations of wheel-slip and tractive/braking force. The input signal is the torque applied to the wheel.

The dynamic equation for the angular motion of the wheel is

\[ I \ddot{\omega} = T - f_w \omega - rF \]  \hspace{1cm} (1)

where \( I \) is the moment of inertia of the wheel, \( f_w \) is the viscous rotational friction and \( r \) is the radius of the wheel. The applied torque \( T \) results from the difference between the shaft torque from the engine and the break torque. \( F \) is the tire tractive/braking force which result from the deformation of the tire at the contact patch.

The vehicle motion is governed by the following equation

\[ m \ddot{v} = F - C_x v^2 \]  \hspace{1cm} (2)

where \( C_x \) is the aerodynamic drag coefficient and \( m \) is the vehicle mass.

The tractive (braking) force (produced on the tire/road interface when a driving (braking) torque is applied to a pneumatic tire) is in the opposed direction of relative motion between the tire and road surface. This relative motion determines the tire slip properties. Slipping is due to deflection in the contact patch [7,8]. Thus wheel-slip \( s \) is associated with the development of tractive or braking force and we can express \( F \) as:

\[ F = f(s) \]  \hspace{1cm} (3)

The wheel-slip is generally described by a cinematic relationship as follow:

\[ \begin{align*}
\vec{s} &= \frac{v_s}{v} & \text{during braking phase} \\
\vec{s} &= \frac{v_s}{ro} & \text{during acceleration phase}
\end{align*} \]  \hspace{1cm} (4)

where \( v_s=v-r\omega_0 \) represents the slip velocity in the contact patch.

Recently many advanced studies [15,22] analyze the behavior of the tire properties in rapid transient maneuver such as cornering on uneven roads, braking torque variation and oscillatory steering. These studies deal with transients in tire force and use the concept of the relaxation length to account the deformation of the carcass, in the contact patch, that is responsible of the lag in the response to lateral and longitudinal slip. The motivation was to improve understanding of the tire behavior with respect to experimental results and then include it in vehicle dynamic simulations. The concept of relaxation length has been formulated particularly for the lateral dynamic to model transient tire behavior (see references: [11,12,14]). This concept has been used also for longitudinal dynamic. In [12,24] the authors have used the relaxation length concept to describe the longitudinal and lateral forces and to study the tire dynamic behavior which is represented by a rigid ring model. In [2,6] the authors give a review to the concept of relaxation length and present a formulation for both longitudinal slip and slip angle as state variables which will be used with any semi-empirical tire model. The wheel-slip can be presented by a first order relaxation length as follows:

\[ \begin{align*}
\vec{s} &= \frac{v_s}{v} & \text{during braking phase} \\
\vec{s} &= \frac{v_s}{ro} & \text{during acceleration phase}
\end{align*} \]  \hspace{1cm} (5)

The steady-state solution of equation (5) equals the normal definition of wheel-slip given by equation (4). The relaxation length \( \sigma \) equals the slip stiffness \( C_x \) divided by the longitudinal stiffness in the contact patch (\( k_x = 2k x \ell^2 \)):

\[ \sigma = \frac{C_x}{k_x} \]  \hspace{1cm} (6)

where \( \ell \) is the half contact length and \( k_x \) is the stiffness per unit of length of the tread.

The slip stiffness is defined as the local derivative of the stationary slip-tire force characteristic (see figure 1):

\[ C_x = \frac{\partial F}{\partial s} \]  \hspace{1cm} (7)

According to equations (3) and (7) and with regard to the characteristic of figure 1, we can suggest a formulation of the time derivative of \( F \) as:

\[ \dot{F} = C_x \dot{s} \]  \hspace{1cm} (8)

During braking phase, the equation (8) becomes then:

\[ \sigma \dot{F} = C_x (-vs + v_s) \]  \hspace{1cm} (9)

1 The upper bar is introduced here to denote the steady-state of wheel-slip with respect to transient slip.
However at small slips, we can write:

$$F = C_\kappa s$$  \hspace{1cm} (10)

with

$$C_\kappa = \left. \frac{\partial F}{\partial s} \right|_{s=0} = 2k_x \ell^2$$  \hspace{1cm} (11)

and let us define $s^d$ as the constant desired wheel-slip. This leads to the desired slip velocity $v_s^d$ defined by:

$$v_s^d = v_s$$  \hspace{1cm} (15)

Let $e_s = s - s^d$ and $e_{vs} = v_s - v_s^d$ denote respectively the tracking errors on wheel-slip and slip velocity, then the wheel-slip error dynamic equation is given by:

$$\sigma \dot{e}_s + v e_s = e_{vs}$$  \hspace{1cm} (16)

So, both $\sigma$ and $v$ are strictly positive in braking phase then equation [16] is exponentially stable and we can prove that:

if $e_{vs} \to 0$ when $t \to \infty$ then $\lim_{t \to \infty} e_s = 0$

In order to study the convergence of $e_{vs}$, we must rewrite the system dynamics in state space form. This is possible by choosing convenient state variables. These are:

$$x_1 = \omega, \quad x_2 = v, \quad x_3 = F$$

The unknown parameters of the system are:

$$\theta_1 = \frac{F_0}{\sigma}, \quad \theta_2 = \frac{1}{\sigma}$$

Then, the state space form of the system can be written:

$$\begin{align*}
x &= f(x) + bu \\
y &= h(x)
\end{align*}$$  \hspace{1cm} (17)

with

$$f(x) = \begin{bmatrix} - \frac{f_w}{l} x_1 - \frac{r}{l} x_3 \\
\frac{C_\kappa}{m} (r x_1 + x_2)^2 + \frac{f_w}{l} x_1 + \left( \frac{1}{m} + \frac{r^2}{l} \right) x_3 \\
k \kappa x_2 + \theta_1 (r x_1 + x_2) - \theta_2 (r x_1 + x_2) x_3
\end{bmatrix}$$

$$b = \begin{bmatrix} \frac{1}{l} \\
\frac{r}{l} \\
0
\end{bmatrix}$$

$$h(x) = \begin{bmatrix} 0 \\
0 \\
x_1
\end{bmatrix}$$

$$u = T$$

at small slip $F_0 = 0$. 

**CONTROL FORMULATION**

For wheel-slip regulation, passive systems approach and sliding mode technique are used. Recall the slip dynamic equation during acceleration phase:

$$\sigma \dot{s} + v_s = v_s$$  \hspace{1cm} (14)
For this system, only $x_1$ is measured (i.e., $y = x_1$), thus the remaining state variables can be estimated by an adaptive observer as will be shown. Let $\hat{x}$ be the state estimation and $\tilde{x}$ the estimation error.

The desired slip velocity ($v^d_s = x^d_2$) can be rewritten as:

$$x_2^d = s^d (rx_1 + x_2)$$
$$x_2^d = s^d (rx_1 + \tilde{x}_2) + s^d \tilde{x}_2$$

with $x_2 = \tilde{x}_2 + \bar{x}_2$. Now if we define $\hat{x}_2^d = s^d (rx_1 + \hat{x}_2)$ and $\hat{e}_v = \hat{x}_2 - \hat{x}_2^d$, then the error $e_v$ becomes:

$$e_v = \hat{e}_v + (1 - s^d) \tilde{x}_2 \quad (18)$$

Then the convergence of $e_v$ will be guaranteed by the convergence of both $\hat{e}_v$, and $\tilde{x}_2$. Before control, state observation have to be designed.

**SYSTEM OBSERVABILITY**

The observability of the system given in (17) is shown by use of rank criterion [10] based on the Lie derivative, (this concept gives a local observability). The Lie derivative of the output $h$ along the field vectors $f$ is given by:

$$L_f h(x) = \frac{\partial h}{\partial x} f(x)$$

The construction of the observability matrix is given by repeated Lie derivative as follows:

$$O(x) = \begin{bmatrix}
\frac{dh}{d(L_f(h))} \\
\vdots \\
\frac{d^{n-1}(L_f(h))}{d^n(L_f(h))}
\end{bmatrix}$$

where $n$ is the dimension of the system state space ($n=3$).

For (17) we obtain the following observability matrix:

$$O = \begin{bmatrix}
1 & 0 & 0 \\
-\frac{f_u}{I} & 0 & \frac{r}{I} \\
\frac{f_u}{I} - r^2 I(\theta_1 - \theta_2 x_1) & \frac{r(\kappa_s + \theta_1 - \theta_2 x_1)}{I} & \frac{(f_u - I\theta_1 (rx_1 + x_2))}{I}
\end{bmatrix}$$

We can see that if the slip ratio is not equal to one ($s<1$) therefore the observability matrix is full rank for any state. Then system (17) is locally observable everywhere in the phase space.

**NONLINEAR OBSERVER DESIGN**

From (17) we can see that nonlinear system is linear with regard to the unknown parameters and it can be rewritten as:

$$\begin{cases}
\dot{x} = \xi(x) + \Psi(x) \theta + gu \\
y = h(x)
\end{cases} \quad (19)$$

with

$$\xi(x) = \begin{bmatrix}
\frac{f_u}{I} x_1 - \frac{r}{I} x_3 \\
\frac{r(\kappa_s + \theta_1 - \theta_2 x_1)}{I} \\
\frac{(f_u - I\theta_1 (rx_1 + x_2))}{I}
\end{bmatrix}$$

$$\Psi(x) = \begin{bmatrix}
0 & 0 \\
0 & 0 \\
rx_1 + x_2 - (rx_1 + x_2) x_3
\end{bmatrix} \quad \theta = \begin{bmatrix}
\theta_1 \\
\theta_2
\end{bmatrix}$$

Then the unknown parameters can be estimated. To estimate both the system state and the unknown parameters we propose the following adaptive and robust observer based on sliding mode approach:

$$\begin{cases}
\dot{\hat{x}} = \hat{\xi}(x) + \Psi(\hat{x}) \hat{\theta} + gu + \Lambda, \\
\dot{\hat{\theta}} = \eta
\end{cases} \quad (20)$$

with

$$\hat{\xi} = \begin{bmatrix}
\frac{f_u}{I} x_1 - \frac{r}{I} x_3 \\
\frac{r(\kappa_s + \theta_1 - \theta_2 x_1)}{I} \\
\frac{(f_u - I\theta_1 (rx_1 + x_2))}{I}
\end{bmatrix}$$

$$\Psi(x) = \begin{bmatrix}
0 & 0 \\
0 & 0 \\
rx_1 + x_2 - (rx_1 + x_2) x_3
\end{bmatrix} \quad \hat{\theta} = \begin{bmatrix}
\hat{\theta}_1 \\
\hat{\theta}_2
\end{bmatrix}$$

$$\Lambda = \begin{bmatrix}
\lambda_1 \\
\lambda_2 \\
\lambda_3 \cdot \text{sign}(x_1 - \hat{x}_1)
\end{bmatrix} \quad \eta = \begin{bmatrix}
\eta_1 \\
\eta_2
\end{bmatrix}$$

where $\dot{x}^T = (\dot{x}_1, \dot{x}_2, \dot{x}_3)$ represents the estimation of state variables and $(\hat{\theta}_1, \hat{\theta}_2)$ are the parameters estimation. The observer gains $\lambda_i$ and variables $(\eta_1, \eta_2)$ will be defined by the convergence analysis hereafter. Recall that only measurement of $x_1$ is available.

The dynamics of the state observation and parameters estimation errors $\tilde{x} = x - \hat{x}$ and $\tilde{\theta} = \theta - \hat{\theta}$ are then:
\[ \dot{x} = \xi + \Psi(x)\theta - \Psi(\xi)\hat{\theta} - \Lambda \]  
\[ \dot{\hat{\theta}} = -\eta \]  
(21)

with

\[ \xi = \left[ \begin{array}{c} -\frac{r}{I}x_3 - \lambda_1 \text{sign}(\bar{x}_1) \\ \frac{2C}{m}(rx_1 + \hat{x}_2)x_2 + \left(\frac{1}{m} + \frac{r^2}{I}\right)x_3 - \frac{C}{m}\bar{x}_2^2 \\ \kappa_{x}\bar{x}_2 \end{array} \right] \]

and where

\[ \Psi(x)\theta - \Psi(\xi)\hat{\theta} = \Psi(\xi)\hat{\theta} + (\Psi(x) - \Psi(\xi))\theta \]  
(22)

Now, consider the sliding surface \( S = \{\bar{x} = 0\} \). The sliding condition that guarantees the attractivity of \( S \) is given by:

\[ \ddot{x}_1 \bar{x}_1 < 0 \]  
(23)

this can be rewritten as:

\[ \ddot{x}_1 - \frac{r}{I}x_3 - \lambda_1 \text{sign}(\bar{x}_1) < 0 \]  
(24)

Inequality (24) is verified if \( \lambda_1 \) is chosen such that

\[ \lambda_1 > \frac{r}{I}\bar{x}_{1,\text{max}} \quad \forall t \in \mathbb{R}_+ \]  
(25)

where \( \bar{x}_{1,\text{max}} \) is the maximum value of \( \bar{x}_1 \) at any time, then \( \dot{x}_1 \) will converge to \( x_1 \) in finite time and stay equal to the actual value for all \( t > t_0 \). Moreover, we have:

\[ \ddot{x}_1 = 0 \quad \forall t > t_0 \]

On the sliding surface, we can consider the mean value of the signal generated by the "sign" function [20]:

\[ \overline{\text{sign}(\bar{x}_1)} = -\frac{r}{\lambda_1} \bar{x}_3 \]  
(26)

By substituting the equivalent form of "sign" in equations (21) and developing with equations (22), we obtain the following reduced sliding dynamics:

\[ \dot{\bar{x}} = -\chi(\bar{x}, \bar{x}) + \overline{\Psi(\bar{x})\hat{\theta}} - \Theta \bar{x} \]  
\[ \dot{\hat{\theta}} = -\eta \]  
(27)

with

\[ \chi(\bar{x}) = \left[ \begin{array}{c} \frac{2C}{m}(rx_1 + \hat{x}_2) \\ (\frac{1}{m} + \frac{r^2}{I})x_3 - \frac{C}{m}\bar{x}_2^2 \\ -\frac{r\lambda_1}{\lambda_{x1}} \end{array} \right] \]

\[ \Theta = \left[ \begin{array}{cc} \frac{C}{m}\bar{x}_2 & 0 \\ \theta_{x1}\bar{x}_3 & \theta_{x2}(rx_1 + \hat{x}_2 + \bar{x}_2) \end{array} \right] \]

where \( x_r = (x_2, x_3) \) is the reduced state and \( \overline{\Psi} \) is a submatrix of \( \Psi \) composed by two last rows.

The stability analysis of the reduced observation errors dynamics will be studied in closed-loop with controller.

**SLIDING MODE OBSERVER BASED CONTROL**

To achieve the above control objectives, we establish the dynamic equation of error \( \dot{\varepsilon}_\text{vs} \) as follows:

\[ \dot{\varepsilon}_\text{vs} = \dot{x}_2 - \frac{r}{I}\bar{x}_3 \]  
(28)

with

\[ g(\bar{x}) = -\frac{C}{m}(1 - s^d)(r_1 + \bar{x}_3)^3 + \frac{r_{m1}}{I}y + \left(\frac{1-r^d}{m} + \frac{r^2}{I}\right)x_3 \]

\[ \alpha = rs^d\lambda_1 - (1-s^d)\lambda_2 \]

Then, the controller based on sliding mode approach can be designed as:

\[ u = \frac{1}{r}(g(\bar{x}) + k \text{sign}(\dot{\varepsilon}_\text{vs})) \]  
(29)

where \( k \) is a positive sliding gain to be defined later. Then by using equation (26), the dynamic equation of error \( \dot{\varepsilon}_\text{vs} \) becomes:

\[ \dot{\varepsilon}_\text{vs} = r\alpha\bar{x}_3 - k \text{sign}(\dot{\varepsilon}_\text{vs}) \]  
(30)

The closed-loop analysis is performed on the basis of the reduced system (27) and control error equation (29). Then let us define \( z = (\bar{x}_r^T, \varepsilon_\text{vs}^T) \) as the augmented state vector with as equations:

\[ \dot{\bar{x}} = -\chi(\bar{x}, \bar{x}) + \overline{\Psi(\bar{x})\hat{\theta}} - \Theta \bar{x} \]  
\[ \dot{\hat{\theta}} = -\eta \]  
(31)

Let us, consider the following Lyapunov function:

\[ V(\bar{x}, \hat{\theta}) = \frac{1}{2}\bar{x}_3^T \bar{x}_3 + \frac{1}{2}\hat{\theta}_r^T Q \hat{\theta}_r \]  
(32)
where $Q=\text{diag}(q_1,q_2)$ is a positive diagonal matrix.

The time derivative of $V(\xi, \tilde{\theta})$ is given by:

$$\dot{V}(\xi, \tilde{\theta}) = \ddot{x}_i^T \ddot{x}_i + \dot{\xi}_i \dot{\theta}_i + \dot{\theta}_i^T Q^{-1} \dot{\theta}_i$$

$$\dot{V}(\xi, \tilde{\theta}) = -\ddot{x}_i^T \ddot{x}_i - \ddot{x}_i^T \theta \ddot{\xi}_i + \ddot{\theta}_i^T (Q^{-1} \ddot{\theta}_i - Q^{-1} \eta)$$

$$+ \dot{\xi}_i \left( \frac{r}{I_{\lambda_1}} \alpha \ddot{x}_i - k \text{sign}(\ddot{\theta}_i) \right)$$

Now we define the parameters adaptation law $\eta$ as:

$$\eta = Q(\dot{\theta}_i, \ddot{\xi}_i)$$ (33)

For implementation, we can develop $\eta$ as:

$$\eta = Q(\dot{\theta}_i, \ddot{\xi}_i)^T \Gamma \ddot{\xi}_i$$ (34)

where $\Gamma=(1,1)$.

Note that on sliding surface we have (in the mean):

$$\ddot{x}_i = -\frac{\eta_0}{r} \text{sign}(\ddot{x}_i)$$ (35)

thus averaging $\eta$ can rewritten as:

$$\eta = -\frac{\eta_0}{r} Q(\dot{\theta}_i, \ddot{\xi}_i)^T \Gamma \text{sign}(\ddot{x}_i)$$ (36)

we obtain

$$\dot{V}(\xi, \tilde{\theta}) = \ddot{x}_i^T \ddot{x}_i + \dot{\xi}_i \dot{\theta}_i + \dot{\theta}_i^T Q^{-1} \dot{\theta}_i$$

$$\dot{V}(\xi, \tilde{\theta}) = -\ddot{x}_i^T \ddot{x}_i - \ddot{x}_i^T \theta \ddot{\xi}_i - \ddot{\theta}_i^T \left( \frac{r}{I_{\lambda_1}} \alpha \ddot{x}_i - k \text{sign}(\ddot{\theta}_i) \right)$$

To guarantee the stability in closed-loop, the matrix $\chi$ must be positive definite and the sliding gain $k$ must be chosen such as:

$$k > \frac{r}{I_{\lambda_1}} |\alpha| \left| \ddot{x}_i \right|_{\text{max}}$$

so, by using inequality [25] we can choose $k>|\alpha| \ddot{x}_i$.

Let us define $p_1$, $p_2$ to be the eigenvalues of the matrix $\chi$. Then if we impose $p_1$, $p_2>0$ we obtain the gains $\lambda_1$ and $\lambda_2$ as:

$$\lambda_1 = \frac{4C^2 \left( \dot{x}_i + m \ddot{x}_i \right)^2 - 2C \dot{m} \dot{x}_i + m \ddot{x}_i (p_1 + p_2)}{1 + p_1^2 \left( m \ddot{x}_i - 2C \dot{m} \dot{x}_i \right)}$$

$$\lambda_2 = \frac{\lambda_1 \left( 4C^2 \left( \dot{x}_i + m \ddot{x}_i \right)^2 - 2C \dot{m} \dot{x}_i + m \ddot{x}_i (p_1 + p_2) \right) - k x_i \left( m \ddot{x}_i - 2C \dot{m} \dot{x}_i \right)}{\kappa r m \ddot{x}_i - \kappa r m \ddot{x}_i}$$

$$\lambda_1 = \frac{\lambda_1 \left( 2C \left( \dot{x}_i + m \ddot{x}_i \right) - m (p_1 + p_2) \right)}{mr}$$ (37)

Now define $\rho_0$ to be the minimum eigenvalue of the matrix $\Theta$:

$$\rho_0 = \lambda_{\text{min}}(\Theta) = \min \left\{ \frac{C y_2}{m}, \theta_1 (x_1 + x_2) \right\}$$ (38)

then, time derivative of $V(\xi, \tilde{\theta})$ can be bounded as follows:

$$\dot{V}(\xi, \tilde{\theta}) \leq -(p_m + \rho_0) \left| \ddot{x}_i \right| - (k - \alpha) \left| \ddot{\theta}_i \right|$$ (39)

where $p_m$ is the minimum of $\{p_1, p_2\}$.

It is worthwhile to note that the observation and control gains can also be computed in order to have positive eigenvalues. This can be done to obtain wide margins for these gain values.

Thus, by an appropriate choice of $p_1$, $p_2$ and $k$, the time derivative of $V(\xi, \tilde{\theta})$ is strictly negative. Consequently, $\ddot{x}_i$ and $\ddot{\theta}_i$ tend asymptotically to zero. However, the estimation error of the unknown parameters is only bounded.

Finally we conclude that the control error $e_s$ converges to zero.

**SIMULATION RESULTS**

To illustrate performances of the proposed approach of control we consider a one-wheel model (equations (1) and (2)) simulated with "Magic formula" tire model. The considered parameters are listed in table (1).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>M</td>
<td>400</td>
<td>Kg</td>
</tr>
<tr>
<td>I</td>
<td>1.3</td>
<td>Kg/m²</td>
</tr>
<tr>
<td>R</td>
<td>0.3</td>
<td>M</td>
</tr>
<tr>
<td>Cx</td>
<td>0.3</td>
<td>N/m²s²</td>
</tr>
</tbody>
</table>

Table 1: The parameters of the tire model

Figure 2 presents the slip-road/friction ($\mu$) characteristic used to generate the tire force to be estimated.

The observer gains are: $\lambda_1=150$, $q_1=55$, $q_2=3.5 \times 10^3$, $p_1=250$ and $p_2=250$. The controller gain is $K=15$. We have taken for control and estimation, an error of 25% on the parameters listed in table (1) and an additional perturbation (white noise) is added to the measured angular velocity. The value of the longitudinal stiffness in the contact patch $\kappa_x$ is chosen by taking the typical values for $k_x$ and $(k_x=17 \times 10^3$ N/m² and $=0.05$ m for a load $F_z=4000$ N). The unknown parameters $\theta_1$ and $\theta_2$ are initialized respectively at $-2 \times 10^3$ N/m⁻¹ and $20$ m⁻¹. The initial conditions are $x_1(0)=y(0)=100$ rad/s, $x_2(0)=0.1$ m/s.
and \( x_3(0) = -2.10^3 \text{N}. \) Figure (3) show the convergence of the angular wheel velocity estimation to actual value in finite time. In figures (4) and (5) we show respectively the asymptotic convergence of the slip velocity and longitudinal tire force to actual values. In simulation, the actual tire force is generated by “Magic formula” tire model. Figure (6) shows the result of wheel-slip regulation. The control is activated after 0.5s during braking phase and the desired wheel-slip values are 0.1, 0.15 and 0.2 respectively. In figures (3) and (4) we see that the desired slip value is changed at time 0.1, 0.15 and 0.2. The desired wheel-slip is reached in transient time less than 0.1s. The delay to reach the desired wheel-slip can be tuned by adjusting the sliding gains. This can enhance the control performance.

To verify the robustness of sliding mode approach, we have first compared with a simple PD control. In figure (7), we show a comparison of response of both sliding mode approach and PD control with an error of 30% in measurement of brake torque for sliding mode control and exact brake torque for PD control. We have included an error of 25% in the parameters of system. The sliding mode control gives a good tracking of desired wheel-slip in this case. The robustness of the proposed approach is tested with changing the value of the coefficient of tire/road adhesion. In figure (8) we simulate a change in tire/road adhesion. Initially, the peak of the curve \( \mu \) vs. \( s \) is 1.05 (figure 2) and about 1.5s (simulation time) we change this peak to 0.5 and about 2.3s, we switch this value to 1. We see on bottom of figure (7) that this change in tire/road adhesion is balanced by control law.

The performance of the sliding mode based control is satisfactory since the estimation error is minimal for state variables and the control error is damped. The convergence of parameters estimation is not necessary, they are tuned only for convergence of control error.

So we can conclude that the proposed observer and control allow us to tune adequately the slipping and in the same time produce good estimation of contact forces.

**CONCLUSION**

In this paper, we have presented a new method for vehicle traction control. The main contribution is due to the proposed differential equation of the tractive/braking force derived from the relaxation length concept. The interest of the proposed model is to be able to estimate the tire force on-line by using only angular wheel velocity measurement. Then this information is used adequately in the design of control. Passive systems approach and sliding mode technique are used for wheel-slip regulation. We give here details of the proposed technique only for braking phase, so a similar development is possible for acceleration phase. Simulation results illustrate the ability of this approach to give good estimation of longitudinal tire force and slip velocity. Further, the regulation of wheel-slip allows to achieve quiet and safe braking. The robustness of the sliding mode control, based on nonlinear observer, versus uncertainties on the model parameters and variation of the tire/road friction has also been shown: the simulation model is different from the one used to design the control law. Further, a comparison of the proposed approach of control with PD control is given. The chattering in the sliding mode control signal can be reduced by using a low-pass filter and performance adjustment (parameters tuning). This work is considered for extension and experimental application of this method for an instrumented vehicle (acceleration and braking).
Figure 4: Slip velocity, actual (dashed), estimation (solid)

Figure 5: Longitudinal tire force, actual (dashed), estimation (solid)

Figure 6: Wheel-slip

Figure 7: Comparison of wheel-slip control, Sliding mode control (dashed), PD control (doted)

Figure 8: Wheel-slip control with changing value of $\mu$ (top), Signal of control SMD (bottom)

REFERENCES


CONTACT

LRP
10-12 Avenue de l'europe, 78140 Vélizy, France.

Phone: (33) 1.39.25.49.70
Fax: (33) 1.39.25.49.70
e-mail: elhadri@robot.uvsq.fr
               msirdi@robot.uvsq.fr