Stability Analysis and Rollover Scenario Prediction
For Tractor Semi-Trailer

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I. Introduction

The tractor semitrailer represents a population of risky vehicles, both for themselves as well as other road users. Moreover, for a limited capacity of the infrastructure, the number and density of heavy vehicles grow faster than those of cars. Accidents involving heavy lorries have serious consequences for road users, and incidents induce major congestions or damage to the environment or the infrastructure at disproportionate economic costs. For instance, the risk of having a dead people is 2.4 higher when lorries are involved in accident [1].

To reduce the number of accidents and to improve safety, several solutions have been studied in some programs dealing with the idea and activity of Intelligent Transportation Systems (US NAHSC program, California PATH Program, Japan’s AHSRA, European programs: ADASE, REPONSE and CHAUFEUR-driven, French PREDIT and ARCOS programs, etc.) [2]. Some orientations of these programs may consist of driver assistance, active safety systems (lateral control) and passive safety systems (detection and warning message under hazardous conditions).

Some commercial systems installed in the infrastructure before a dangerous cornering, are able to measure the truck speed. In case of overspeed leading to a rollover in the cornering, a warning is sent to the driver in order to incite him to decrease its speed. Other systems are installed aboard the truck. They can, using informative (passive) nature as measuring lateral acceleration, issue a warning signal to the driver when it goes beyond some risky thresholds. In case of active systems, the concept is to minimize the lateral acceleration by braking action, steering action, suspension action or anti-roll action or a combination of all.

Naleck and al [3] have defined, for the detection of rollover, a measurement based on the energy reserve until to have a rollover situation (Rollover Prevention Energy Reserve). At the same time, Dunwoody simulated the steady state cornering performance of a tractor semitrailer fitted with an active roll control system. The control system required the measurement of the trailer lateral acceleration and the relative roll angle between the tractor and the trailer. The study stated that such a system could raise the static rollover threshold by 20-30% [4].

Dahlberg gave the definition of the remaining energy margin that lead to rollover [5]. This margin is the difference between the critical potential energy and the sum of the potential and kinetic rollover energy.

Chen et al. defined a short predictive criterion called TTR (Time to Rollover) which takes into account the dynamic state evolution of the vehicle [6].

This work is developed as part of the "safety of heavy vehicle" subject of ARCOS 2004 program. The objectives are to develop a strategy to detect critical situations and trigger a warning signal under dangerous conditions such as unstable yaw or rollover, and allow real-time detection
of essential factors leading to vehicle instability or influencing vehicle dynamic performance.

This paper is organized as follows: in section II, we describe the simplified heavy vehicle model. Section III describes the rollover model. In section IV, we formulate the stability analysis of heavy vehicle. In section V, we describe the prediction rollover and draw a conclusion in section VI.

II. HEAVY VEHICLE MODEL

A. Coordinate System

The type of heavy vehicle considered in this work is a tractor-semi-trailer with 5-axles (2-axles for tractor and 3-axles for semi-trailer). In order to present the essential dynamics in cornering vehicle manoeuvre, we adopt a vehicle configuration representing a two-axle tractor with one chassis body, and semi-trailer with one wheel axle and one chassis [11]. In this configuration, we consider only the dynamics of the two bodies (i.e. tractor and semi-trailer’s chassis). The roll angle is considered around the tractor roll axis. The pitch and bounce are neglected in this model.

Four coordinate systems are used to describe the motions of the vehicle sprung and unsprung masses as shown in figure 1.

![Coordinate axis system and model structure](image)

Fig. 1. Coordinate axis system and model structure

To develop the dynamics equations of the model, we consider the motion of the two sprung masses in the coordinate system, see figure 1. \((X_EY_EZ_E)\) is the earth-fixed coordinate system. \((X_iY_iZ_i)\) and \((X_uY_uZ_u)\) are the tractor and semi-trailer’s sprung masses coordinate systems respectively, tied to the epicenter of each body. \((X_uY_uZ_u)\) is the tractor’s unsprung mass coordinate defined at center plane of the front axle with \(Z_u\) is parallel to \(Z_E\). The relative motion of \((X_uY_uZ_u)\) with respect to the earth-fixed coordinate system \((X_EY_EZ_E)\) is the translation motion of the tractor in the horizontal plane and its yaw motion along \(Z_E\) axis. The roll motion is described by motion of coordinate \((X_iY_iZ_i)\) relative to the coordinate \((X_uY_uZ_u)\). The articulation angle between the tractor and trailer can be described by relative motion of the coordinate \((X_iY_iZ_i)\) with respect to the coordinate \((X_uY_uZ_u)\).

With this description of the coordinate systems and their relative motion, we consider the following generalized coordinates:

- \(x_E\) : position of the tractor C.G. in \(X\) direction of the earth-fixed coordinate system,
- \(y_E\) : position of the tractor C.G. in \(Y\) direction of the earth-fixed coordinate system,
- \(\psi\) : yaw angle of the tractor,
- \(\phi\) : roll angle,
- \(\psi_f\) : angle between tractor and trailer at fifth wheel.

This vehicle motion description allows the calculation of the translational and rotational velocities of each body-mass at C.G. by study of the kinematics with respect to different reference frames. Then, the total kinetic energy \((E_K)\) and potential energy \((E_P)\) are calculated in the inertial reference frame \(X_EY_EZ_E\). The expressions of kinetic energy and potential energy are used to derive the vehicle model based on the Lagrangian mechanics approach.

B. Suspension model and Tyre model

A large majority of heavy vehicle suspensions use the leaf spring as the vertically compliant element. For the sake of simplicity, we assume that the suspension is modelled as the combination of a nonlinear spring and a damper element. The vertical force acting on the vehicle sprung mass through the suspension system is equal to the static equilibrium force plus the variation force [6], which is denoted as \(F_s\), from the spring equilibrium point, as illustrated in figure 3.

The suspension forces \(F_s\), are obtained by the function of suspension deflection and deflection rate as

\[
F_{si} = F_{eq} + F_{ki} (e_i) + F_{di} (\dot{e}_i)
\]

where \(F_{eq}\) is the equilibrium force, \(F_{ki}\) is the spring force, \(F_{di}\) is the damping force, and \(i = 1, \ldots, 6\).

Vertical deflections of the suspension are obtained by derivation versus time \(\frac{d}{dt}\).

Neglecting masses of unsprung bodies, tire normal forces in the \((X_uY_uZ_u)\) coordinate system before the tire lift-off are obtained by static analysis as follows. After the tire lift-off, the tire normal forces are obtained using static analysis and the fact that the normal force of the lifted tires is null.

Longitudinal and lateral forces of tires are obtained using the tire model [10]. The parameters of tire are obtained by PROSPER simulator of vehicle dynamics. The tire longitudinal/lateral forces can be decomposed in the \((X_uY_uZ_u)\) coordinate system as shown in figure 2.

C. Equation of Motion

The previous description of the vehicle motion allows the calculation of the translational and rotational velocities of each body-mass at C.G. by the study of the kinematics with respect to different references frames [11]. Then the total kinetic energy \((E_K)\) and potential energy \((E_P)\) are
calculated in the inertial reference frame \((X_EY_EQE)\). The expressions of kinetic energy and potential energy are used to derive the vehicle model based on the Lagrangian mechanics approach. By using the Lagrange's equation
\[
\frac{d}{dt} \left( \frac{\partial E_K}{\partial \dot{q}_i} \right) - \frac{\partial E_K}{\partial q_i} + \frac{\partial E_P}{\partial q_i} = F_g, \quad (3)
\]
we obtain the model of the vehicle in the following form:
\[
M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) = F_g \quad (4)
\]
where \(q_i\) is the \(i^{th}\) generalized coordinate and \(q\) is the generalized coordinate vector defined as \(q = [x, y, \psi, \phi, \psi_f]\). \(F_g\) represents the vector of generalized forces. \(M\) is the inertial matrix that is symmetric positive definite, \(C(q, \dot{q})\dot{q}\) is the Coriolis and Centrifugal forces and \(G\) is the gravity vector.

The generalized forces notice by vector \(F_g\) represents the effect of the external forces acting on the vehicle body. These forces are from the tire-road interface and suspensions defined in terms of the longitudinal and lateral tire forces and vertical forces.

### III. Description of Rollover Model

Since the vehicle is a dynamic system, we need to study its dynamic behavior. In this part, we present a description of the rollover model with simplification to take into account the tractor semitrailer model. The simplified representation of rollover model is shown in figure 3.

It consist of two subsystems: sprung mass of the tractor semi-trailer dynamics and unsprung mass dynamics, connected by the suspension system. Both masses are connected through the suspension spring and damping elements and through a virtual point, which represents the roll center. This point allows only rotational movements between the masses and permits the sprung mass to rotate relatively to unsprung mass.

During the rollover sequence, it is assumed that the vehicle is in permanent contact with the road.

The chosen rollover system is composed of two rigid bodies and represented as a multi body system.

As can be seen in figure 3, the system can be described in the \(YZ\)-plane. It is assumed to be symmetric to vertical axis which goes through the roll center.

### IV. Stability Analysis of Heavy Vehicle

The rollover statistics on heavy vehicles, show that the majority of accidents have been produced on the countryside rather than in towns. Most of the rollovers occurred during driving situations like braking maneuvers or vehicle skidding. The main influence comes from the vehicle velocity, the road pavement, and the friction change.

#### A. Parameters Influencing

In this part a stability analysis of heavy vehicle due to parameters variation mentioned above is presented.

1) Steering Angle: The steering angle is a parameter strongly related to the mobilization of the lateral forces. In the following experimentation, we varied this angle. Figure 4 shows that the stability margin may be strongly decreased with speed.

2) Adherence Coefficient: Adhesion was one of the most important parameters in the rollover accident. Figure 5 represents the stable and unstable domains. When adhesion decreases, the heavy vehicle is (rollover) stable. The rollover principal characteristic i.e. the strong mobilization of the pneumatic contact is represented: when adhesion is lower than 0.45, which corresponds to a strongly wet roadway, the heavy vehicle does not turn over. However, other problems appear like trajectory loss while slipping.

3) Mass and Height of the Gravity Center: Figure 6 shows the stability domain of heavy vehicle compared of the height of center of gravity and longitudinal speed variations.
Fig. 5. Stability adhesion/speed plane

Fig. 6. Stability height of center of gravity/speed plane

We note that an increase height of CG strongly decreases the maximum speed to which a heavy vehicle can approach to a turn. Therefore the height of CG has a very important influence, in particular for the low height of CG between 0.5m and 1m, critical acceleration passes from $6.3\,\text{m.s}^{-2}$ to $4.3\,\text{m.s}^{-2}$, against $3.4\,\text{m.s}^{-2}$ for 1.5m.

4) Other Factors Influencing Roll Stability: Suspension lash, present in the leaf spring suspension systems commonly used on heavy vehicles, degrades the rollover threshold by reducing the effective roll stiffness of the suspensions.

Torsional compliance of the vehicle frames also reduces the rollover threshold. For example, a flexible trailer frame rolls to a greater angle under the influence of lateral acceleration, thus increasing the magnitude of the destabilizing lateral displacement moment. Furthermore torsional compliance of the tractor frame reduces the ability of the tractor steer axle to provide a stabilizing moment which resist to the roll motion of the payload.

Torsional compliance of the vehicle couplings reduces roll stability in a similar way. Note that the roll stiffness of a conventional fifth wheel coupling decreases with articulation angle, and the roll moment that can be transmitted through a fifth wheel coupling saturates at an included angle of around 2.

V. Prediction of Rollover

A. Prediction of Rollover Threshold Acceleration

In case of heavy vehicle, the rollover evolution can be presented as

1) a) i) roll angle evolution of the sprung mass before wheels lift-off,

ii) lift-off of wheels is due to the variation of the forces applied to the unsprung mass.

Figures 7 and 8 show different forces and moments applied in the sprung and unsprung tractor and semitrailer masses respectively.

In such case, one may consider the following simplification hypotheses:

- suspension deflection is neglected,
- the roll angle is the same for the tractor and semitrailer,
- inclination of the roll axis is neglected and its load is symmetric over the left and right wheels.

The quasi-static roll equations for the sprung and unsprung masses of tractor semi-trailer are

$$\sum M_{iO,A} = 0 \quad (5)$$

The sprung mass equation, see figure 7 is given by

$$-M_{iR} - M_{rR} + (F_{slR} + F_{slR}) \frac{S_2}{2} + P_2h_2\phi_R + \frac{P_2}{g} a_{yR}h_2 = 0 \quad (6)$$

where $a_{yR}$ is the lateral acceleration of the trailer, $M_{iR}, M_{rR}$ are left and right moment respectively, $F_{slR} + F_{slR}$ are left and right suspension forces of trailer, $h_2$ is the height of gravity center and $P_2$ is mass of trailer. Using $M_i = k_{\phi,i}\phi$, where $k_{\phi,i}$ is the suspension roll stiffness. The roll angle of the sprung mass, $\phi_R$, can be expressed in terms of the lateral acceleration as

$$\phi_R = \left( \frac{P_2S_2}{2(k_{\phi,i} + k_{\phi,i}) - P_2h_2} \right) + \left( \frac{P_2h_2}{g((k_{\phi,i} + k_{\phi,i}) - P_2h_2)} \right) a_{yR} \quad (7)$$

equation 7 can rewritten in the form

$$\phi_R = A a_{yR} + B \quad (8)$$

Figure 8 shows the free body diagram for unsprung mass in a left turn.
Fig. 8. Forces and moments applied on the unsprung mass.

The roll moment equilibrium equation of the unsprung mass is given by

\[ M_l R + M_r R + (F_{yL} + F_{yR}) h_2 - L_2 = 0 \] (9)

where \( F_{yL}, F_{yR} \) are left and right lateral forces of trailer respectively.

We consider that the total lateral force is uniformly distributed over the left and the right wheels. Using \( F_{yi} = \frac{P}{2g} a_y R \), the angular momentum of trailer \( L_2 \).

\[ L_2 = (k_{pl} + k_{pr}) B + \left( (k_{pl} + k_{pr}) \frac{P h_2}{g} A \right) a_y R \] (10)

where \( A, B \) are coefficients of equation 8. Since the maximum angular momentum on the axle is \( \frac{P h}{g} \) in a left turn and \( -\frac{P h}{g} \) in a right one, the threshold lateral acceleration at the trailer tire lift-off can be obtained as

\[ a_{ycR} = \frac{\frac{P h}{g} - (k_{pl} + k_{pr}) B}{(k_{pl} + k_{pr}) \frac{P h}{g} A} \] (11)

Signs of absolute value of 11 represent the direction vehicle lateral acceleration (left or right respectively). We consider that \( a_{ycR} \) is the trailer critical lateral acceleration. Figure 9 shows evolution of roll moment compared to lateral acceleration of trailer. For a tractor-semitrailer, the rollover sequence is split into two intervals.

1) trailer roll angle up to a critical value (tire lift-off),
2) tire lift-off critical angle is transmitted to the tractor.

Since the load on the outside wheel of the trailer (in a turn) is nil after the outside wheel lift-off of the trailer, the total suspension roll stiffness after the trailer tire lift-off becomes smaller than before the trailer tire lift-off. In a simplified approach, it is assumed that the trailer-suspension roll moment is kept constant between the left and right wheels after the trailer tire lift-off. The moment equation of tractor is

\[ -M_{lT} - M_{rT} + M_{tc} + P_1 \frac{g}{h_1} a_{y0} + P_1 (y_0 + h_1 (\phi_0 + \phi_T)) + \frac{P_1}{g} h_1 (a_y R + a_y T) = 0 \] (12)

where \( \phi_0 = \phi_R \) is initial roll angle transmitted by trailer, \( \phi = \phi_T + \phi_R \) is the total roll angle of tractor semitrailer and \( a_y = a_y R + a_y T \). From goes equation ??, we obtain

\[ -2k_{pT} \phi_T - 2k_{pR} B + \left( 2k_{pR} \frac{P h_2}{g} A \right) a_y R + P_1 \frac{g}{h_1} a_y = 0 \]

The threshold lateral acceleration at the tractor-semitrailer \( a_y \), is given by

\[ a_y = \frac{\frac{g}{h_1} \left( 2k_{pT} \phi_T + 2k_{pR} B + \left( 2k_{pR} \frac{P h_2}{g} A \right) a_y R \right.}{P_1 \frac{g}{h_1} \left( h_1 \phi_T + h_1 \phi \right)} \] (13)

For a simple approach, it is assumed that the tractor and trailer roll moments are the same. In practice, the semitrailer carries the load, therefore the load undergoes a more important lateral acceleration than the tractor. In this case, one can notice that

B. Prediction simulation results

Fig. 10. Steer Angle Input

Fig. 11. Normal force of trailer

The model is simulated for cornering manoeuvre at constant vehicle speed. In this case, we assume the longitudinal speed to be constant or \( V_x = 85 Km/h \) during
Fig. 12. Lateral Acceleration

Fig. 13. Roll angle

Fig. 14. Load Transfer Ratio

simulation. Figure 10 shows the steering angle input of left and right wheels of tractor.

Figures 11, 12 and 13 show simulation results, with step steering inputs, the evolution of left and right wheels trailer normal forces, lateral acceleration and roll angle respectively. In figure 11, we notice the right wheel force tends to zero, representing wheel lift-off. This will lead to rollover situation. Such case may also be seen in figure 12, where the lateral acceleration has exceeded its critical value, leading by the same, to rollover situation. A similar conclusion can be drawn from the roll angle curve shown in figure 13. Figure 14 shows the presentation of Load Transfer Ratio of the simulated case. One can see that for the given parameters, lateral acceleration represents real image of the vehicle rather than LTR coefficient.

VI. CONCLUSION

In this paper, an approach to evaluate simulation of vehicle lateral acceleration is presented. A representative model of the essential dynamics is developed. We have focused the study on stability analysis of some parameters influence on the dynamic behavior of the heavy vehicle. The prediction of threshold lateral accelerations at the tractor trailer tire lift-off was proposed for a roll warning system. The predicted accelerations were compared with the threshold accelerations obtained by simulation to validate the prediction. However, the predicted accelerations for the tire lift-off have some deviation from the values obtained by rollover coefficient (LTR). This is mainly due to assuming constant suspension roll moment after the tire lift-off. In real implementation, this problem can be solved by applying a safety factor to the threshold acceleration. The following step of this work consists to establish a protocol of test in order to experimentally evaluate the approach.

REFERENCES